Controlling quantum fluctuations with nonlinear interactions

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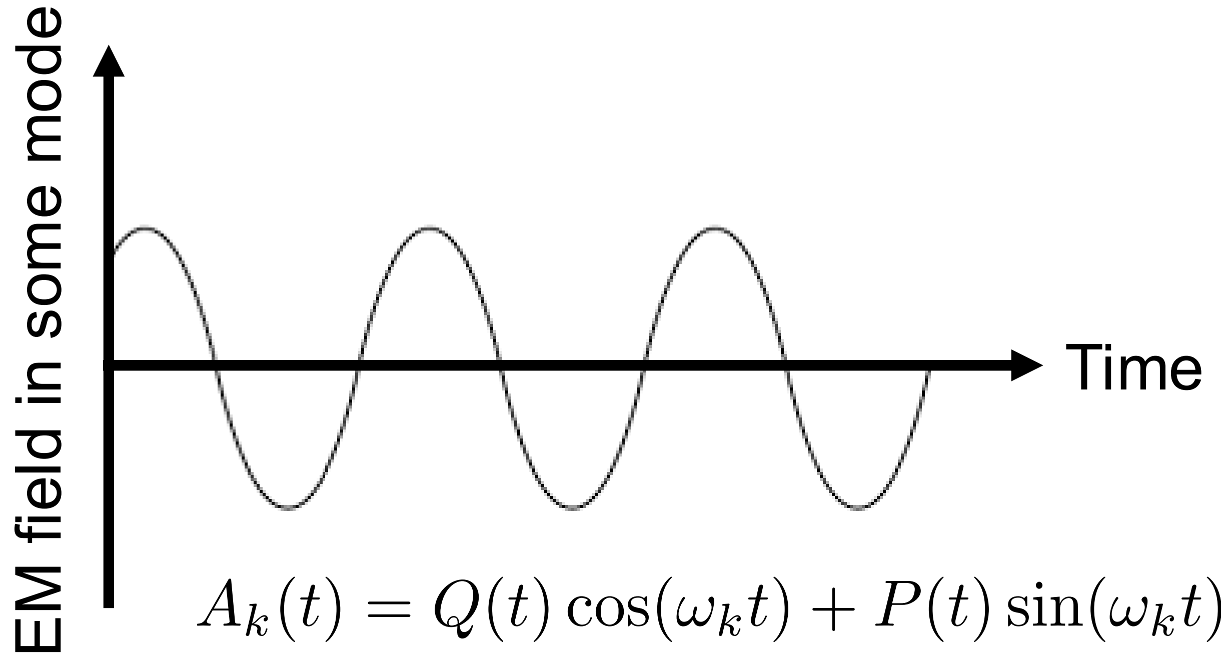
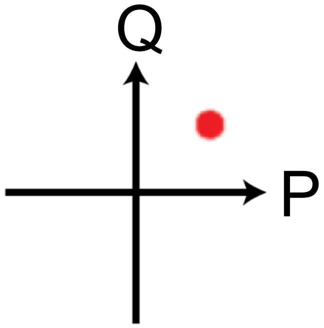
Department of Physics, Harvard University

Quantum fluctuations in electromagnetic fields



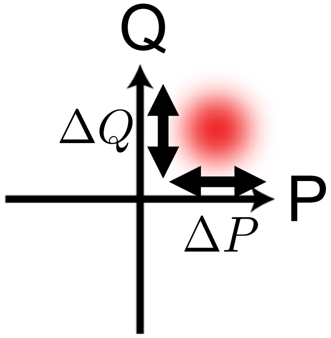
$$A_k(t) = Q(t)\cos(\omega_k t) + P(t)\sin(\omega_k t)$$

Classical states



Q, P can be definite

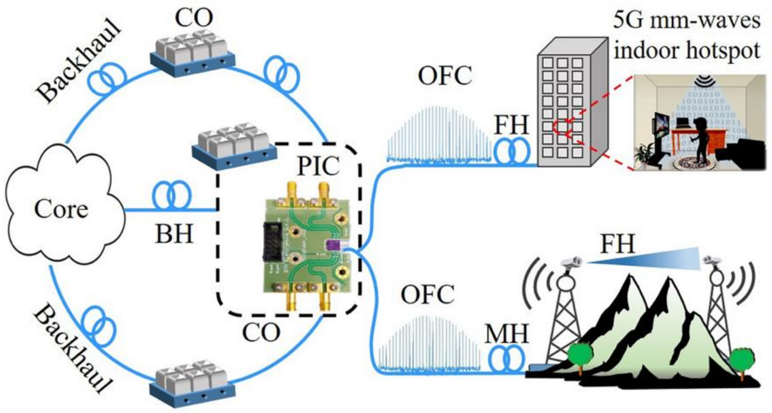
Quantum states: uncertainty



$$\Delta Q \Delta P \geq 1$$

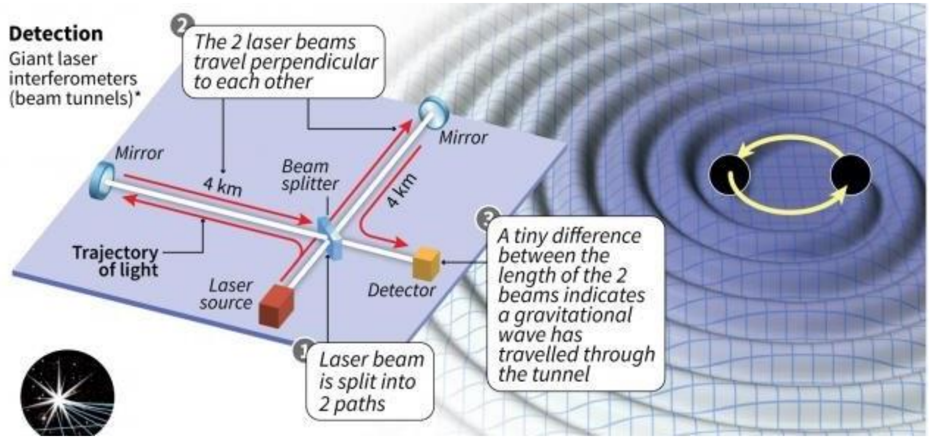
Limits from quantum noise in different domains

Systems limited by quantum and similar noises

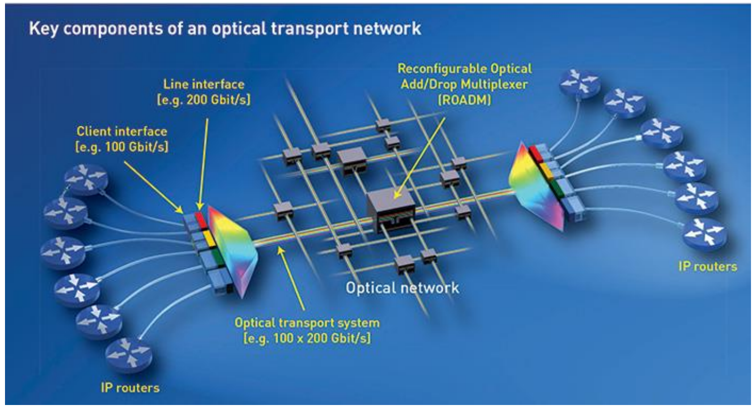


5G/6G networks

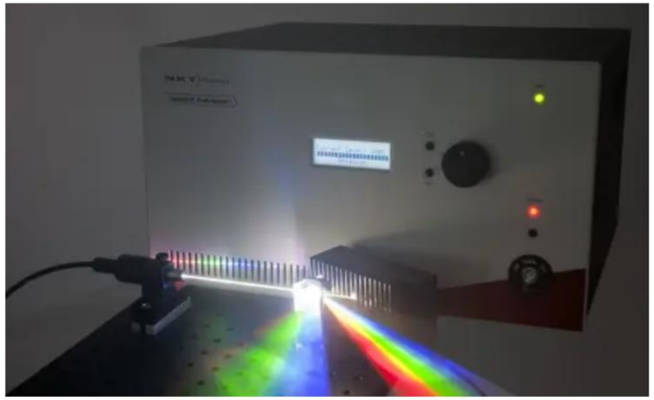
Medicine



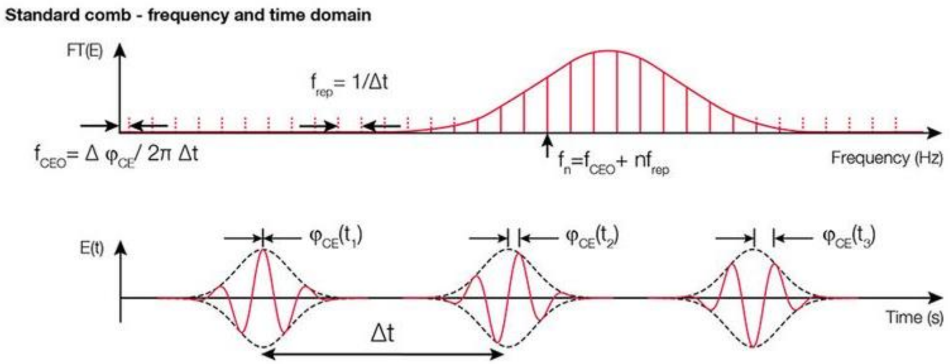
Interferometers



Optical fiber networks

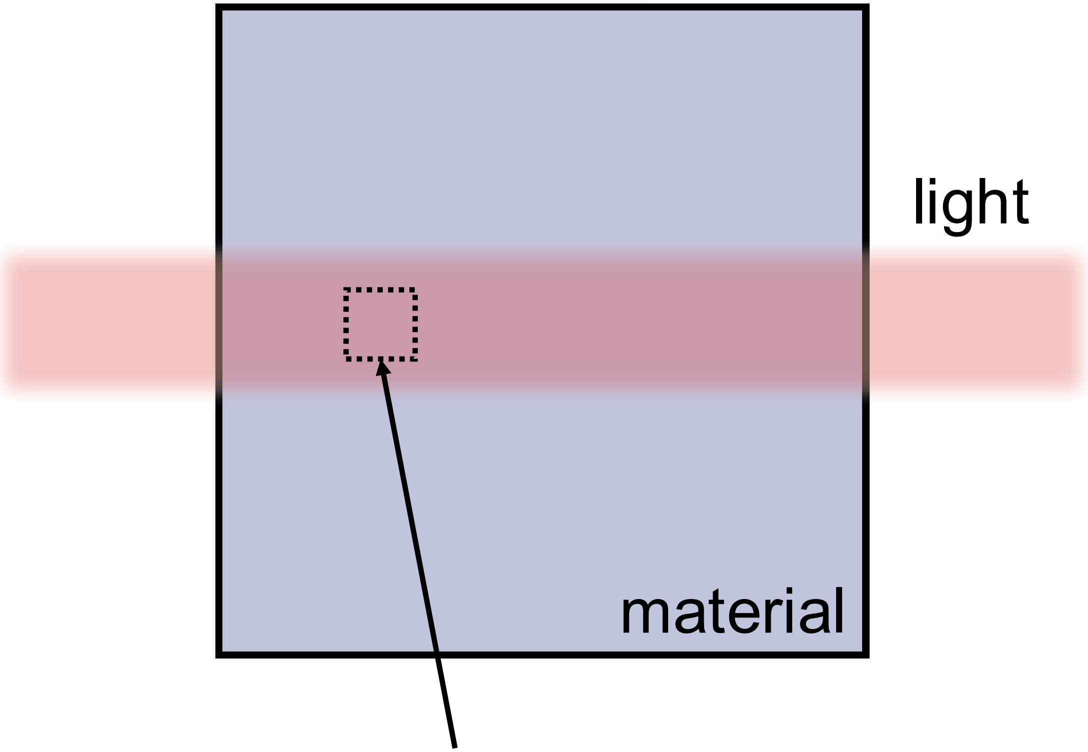


Light sources



Precision measurement

Nonlinear response of matter to light



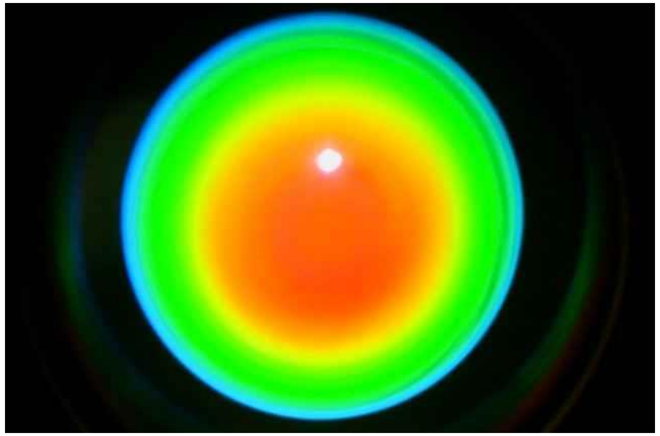
$$P_i = \epsilon_0 \left(\underbrace{\chi^{(1)}_{ij} E_j}_{} + \underbrace{\chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \dots}_{} \right)$$

Material polarization | Weak fields | Stronger driving fields (e.g., using focused lasers)

Manifestations of nonlinear optics

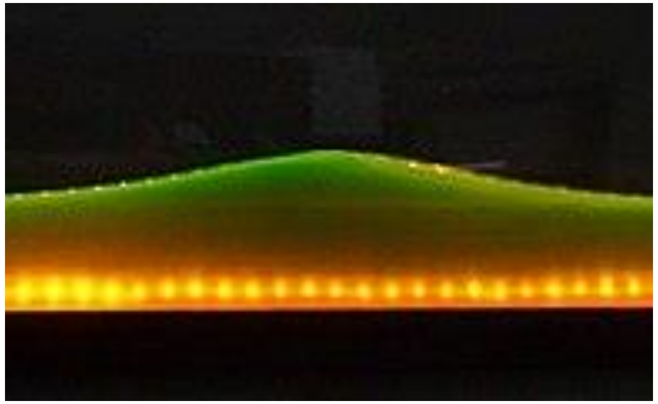
Harmonic generation

Frequency down-conversion

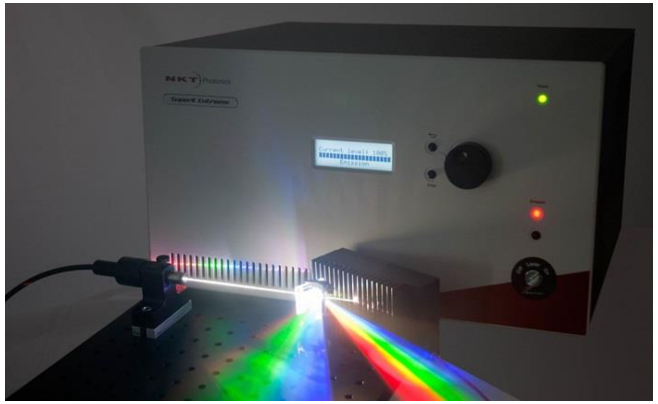


Electro-optic effect

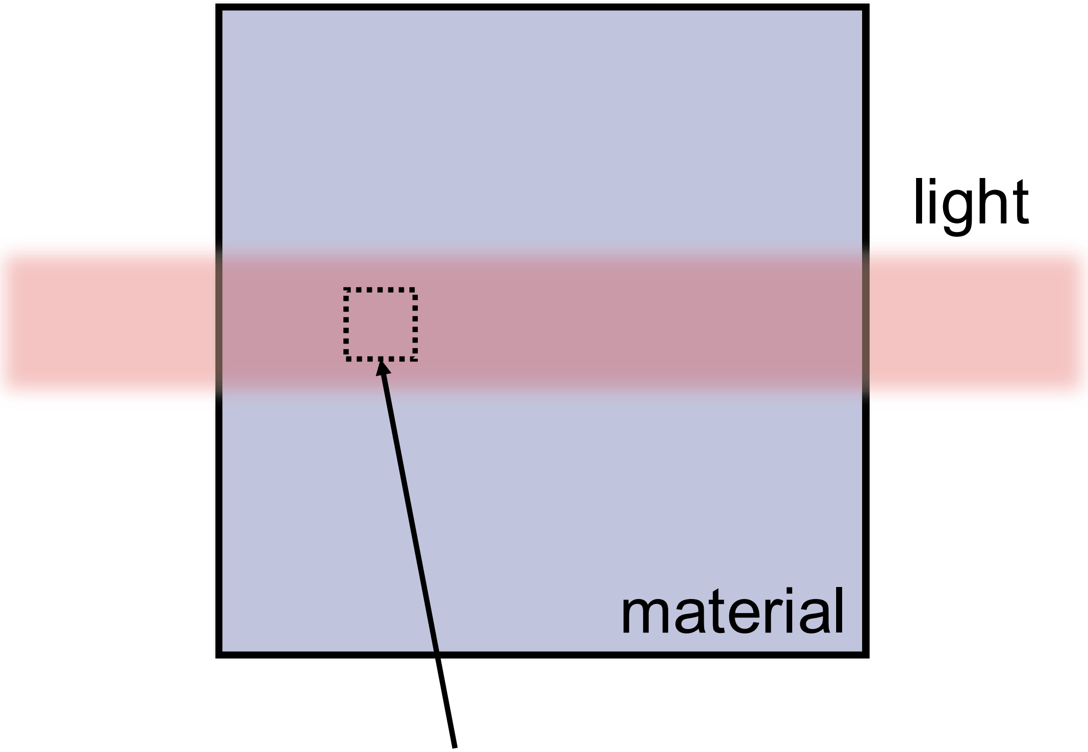
Solitons of light



Supercontinuum generation



The quantum side of nonlinear optics



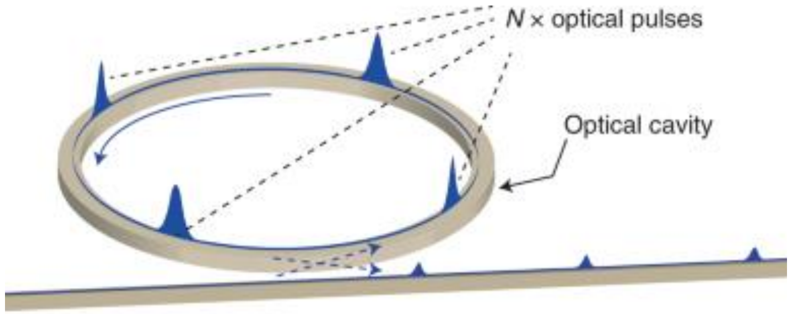
$$H = \int d\mathbf{r}\ \frac{1}{2}\left(\epsilon E^2 + \mu H^2\right) + \eta^{(2)} E^3 + \eta^{(3)} E^4 + \ldots$$

linear optics

photon-photon interactions: quantum states of light like "squeezed states"

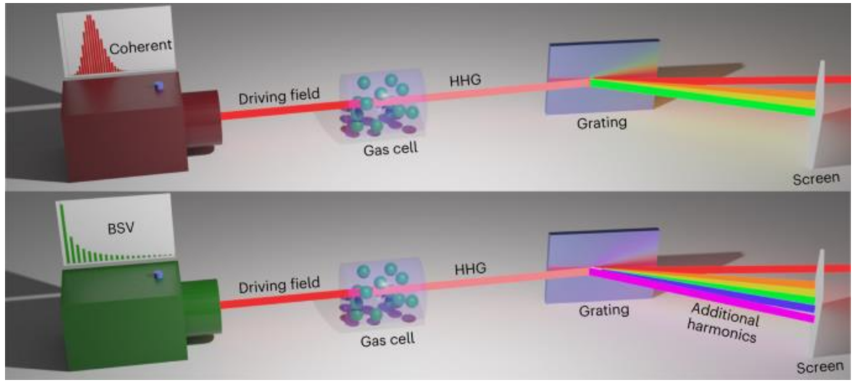
Motivation: quantum nature of complex nonlinear systems

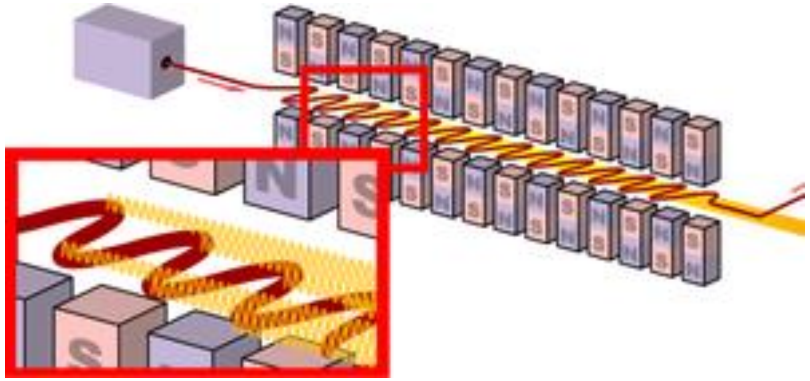
Frequency combs: dissipative solitons



Supercontinuum generation

X-ray sources: High-harmonic generation and free-electron lasers





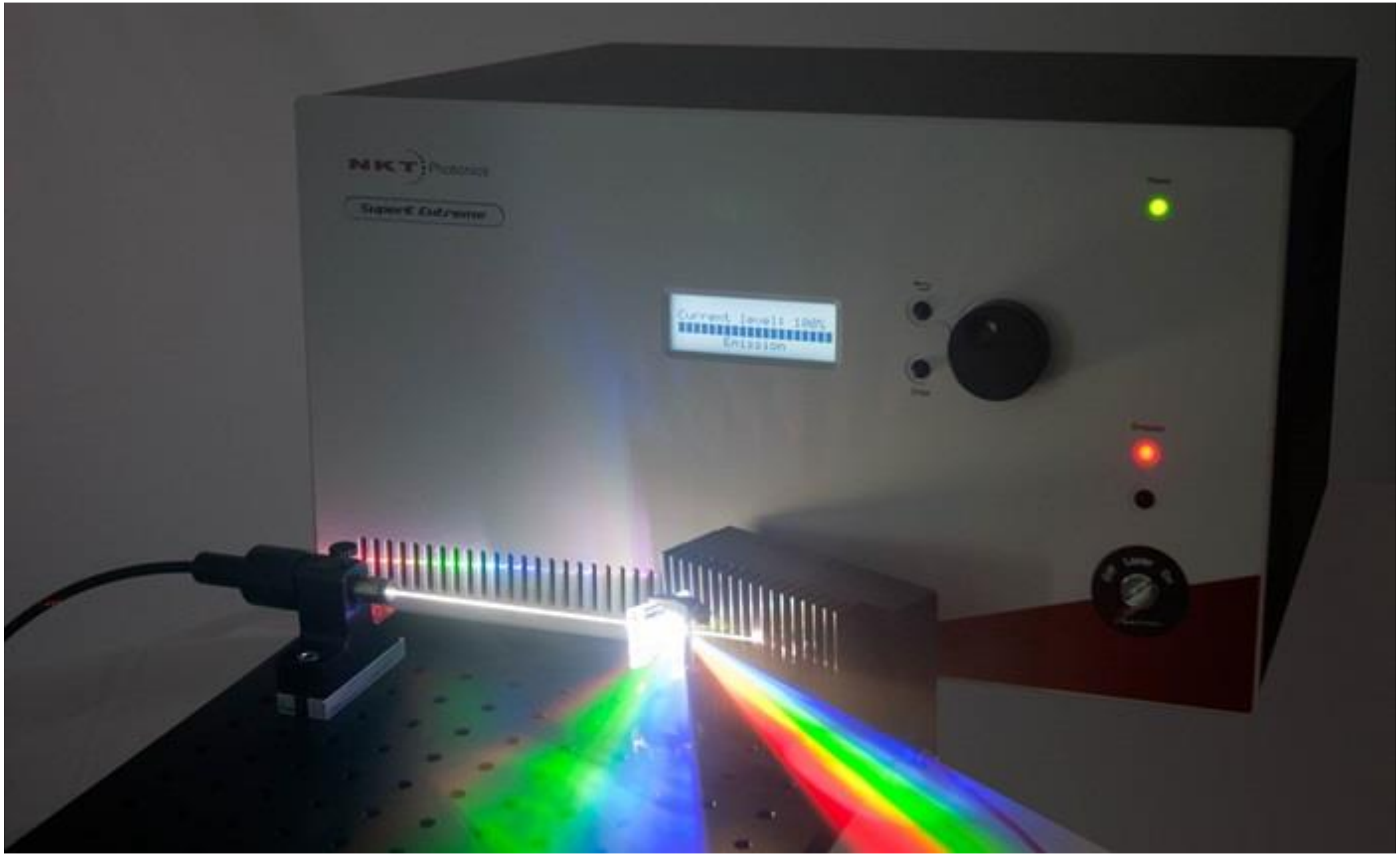
Huge range of phenomena which we understand classically but know little about the quantum optical properties (correlations, noise, etc.) relevant to many current research problems. No framework that directly leverages mature classical understanding!

Shiekh Zia Uddin
(MIT)

The quantum optics of supercontinuum generation

Based on: Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Supercontinuum generation



"White as a lamp, bright as a laser"

The classical description of supercontinuum generation

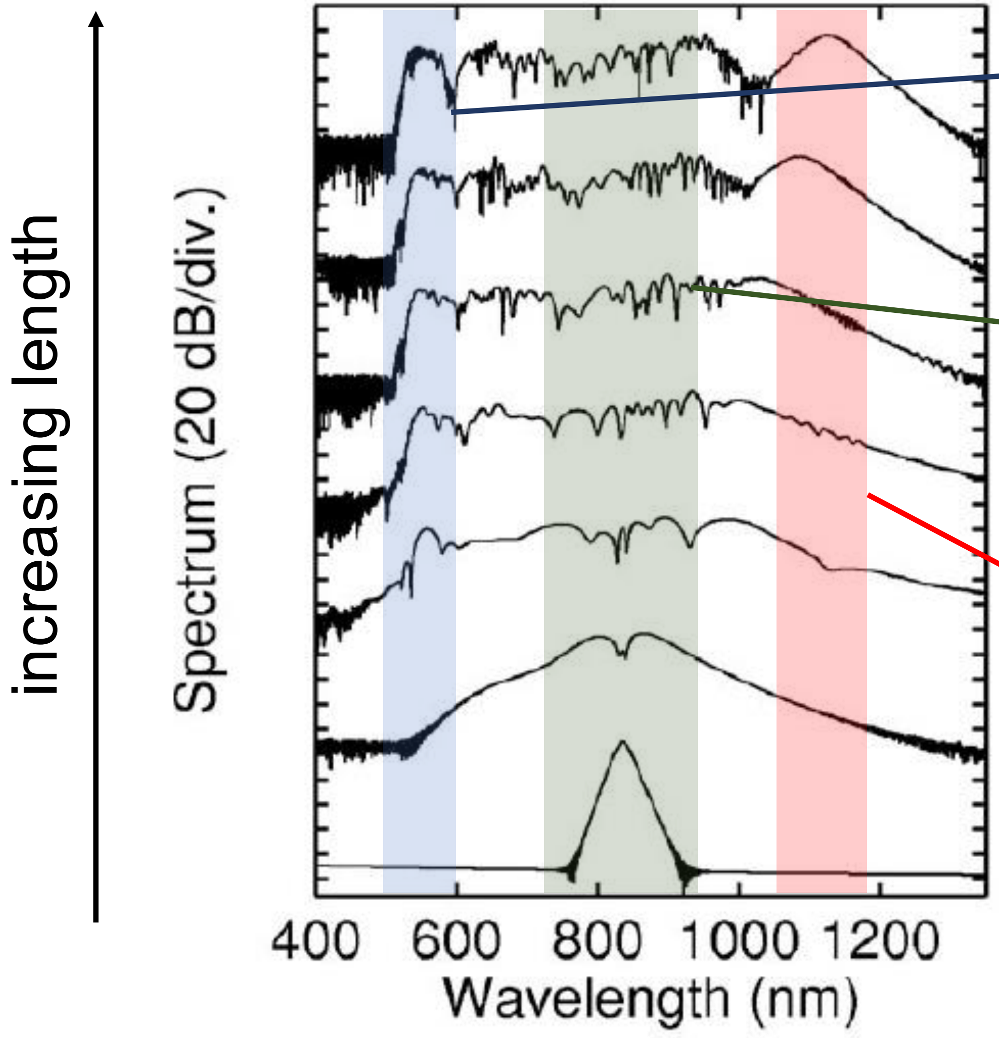
Pulse evolution: the generalized nonlinear Schrodinger equation

$$\partial_z A(z,t) = -\frac{\alpha}{2}A(z,t) + \sum_{k=2}^{\infty}\frac{i^{k+1}\beta_k}{k!}\partial_t^k A(z,t) + i\gamma A(z,t)\int dt' R(t')|A(z,t-t')|^2$$

- loss: $-\frac{\alpha}{2}A(z,t)$
- dispersion: k = k(ω)
- Nonlinearity: instantaneous (electronic) + delayed (phonons)

This equation (+ variants) has been studied by many authors for a long time (50+ years)!

The physical processes leading to supercontinuum generation



high-order dispersion + nonlinearity

instantaneous part of nonlinearity

soliton dynamics (dispersion + instantaneous nonlinearity) and Raman scattering

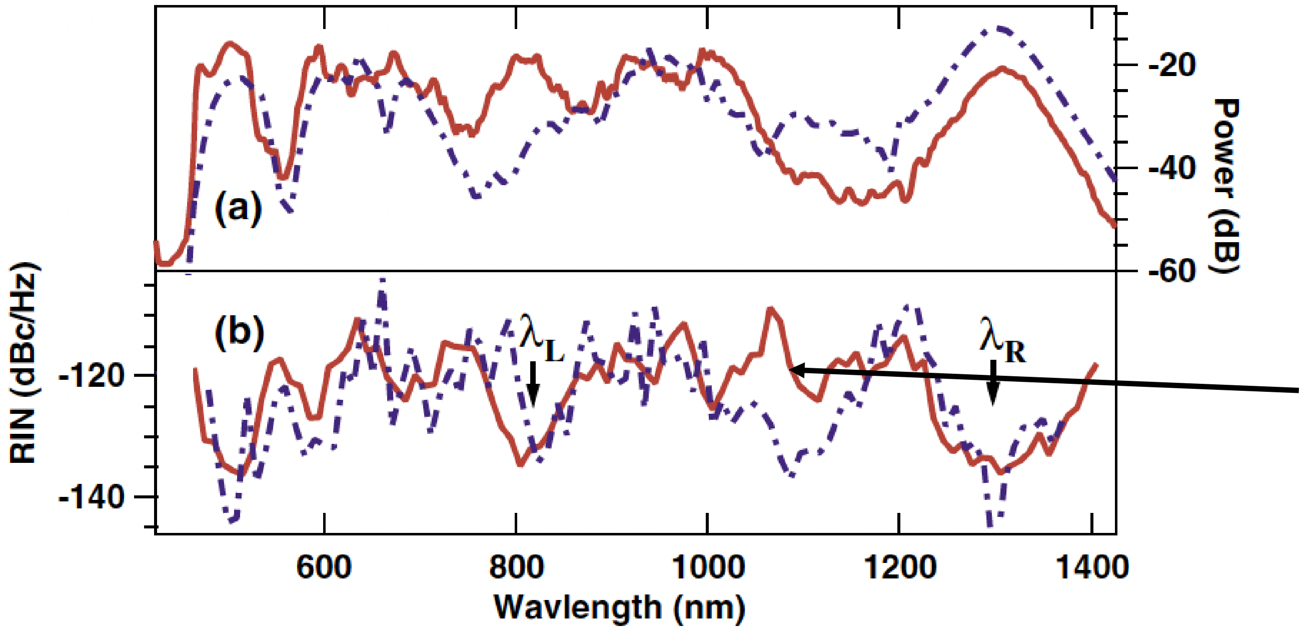
Dudley *et al. Rev. Mod. Phys.* (2006).

Quantum fluctuations in supercontinuum generation

VOLUME 90, NUMBER 11 PHYSICAL REVIEW LETTERS week ending 21 MARCH 2003

Fundamental Noise Limitations to Supercontinuum Generation in Microstructure Fiber

K. L. Corwin,[1] N. R. Newbury,[1] J. M. Dudley,[2] S. Coen,[3] S. A. Diddams,[1] K. Weber,[1] and R. S. Windeler[4]

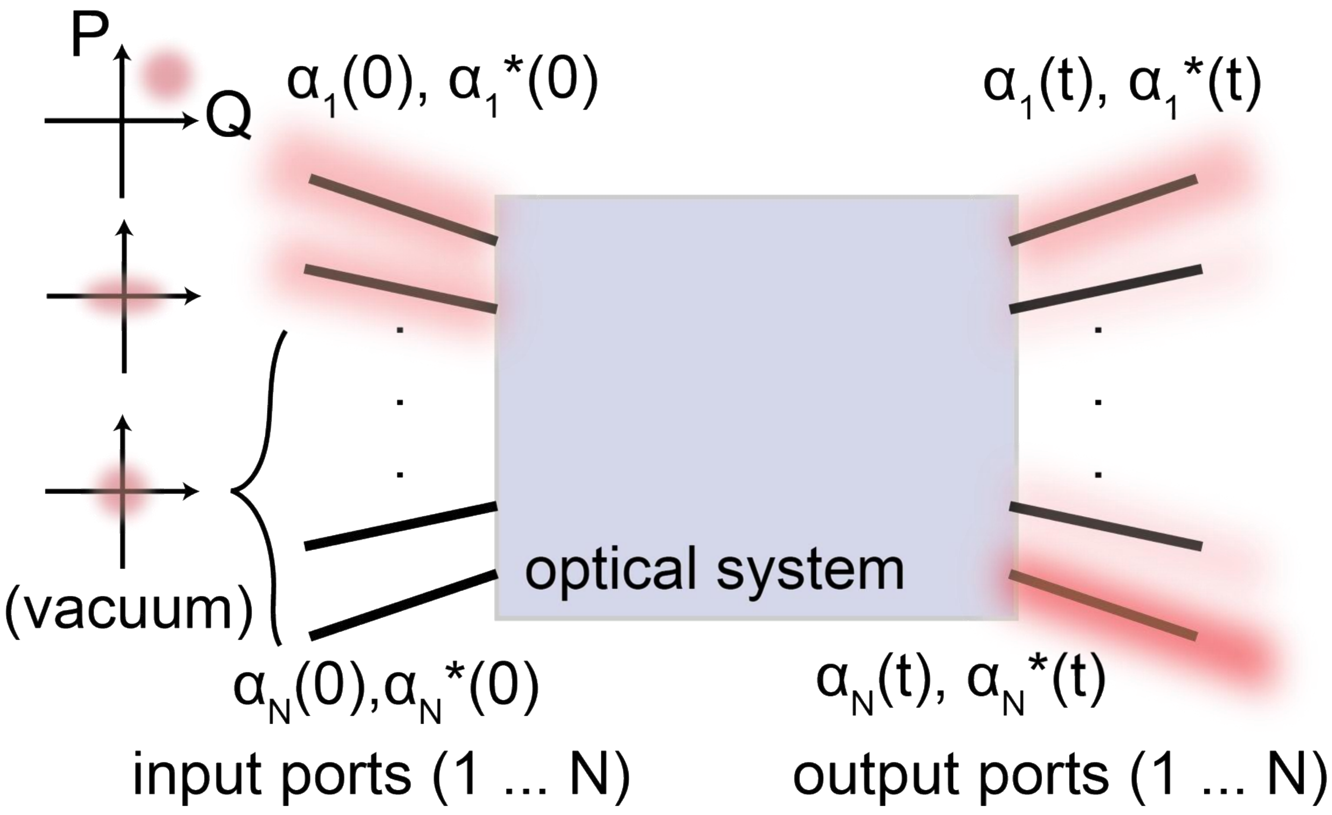


Corwin *et al.* PRL (2003).

Nearly 1-10% pulse-to-pulse fluctuation from inputs with <0.01%. Individual wavelengths are not in coherent states, even when initial light is!

Picture of supercontinuum: fundamentally noisy and noise-sensitive.

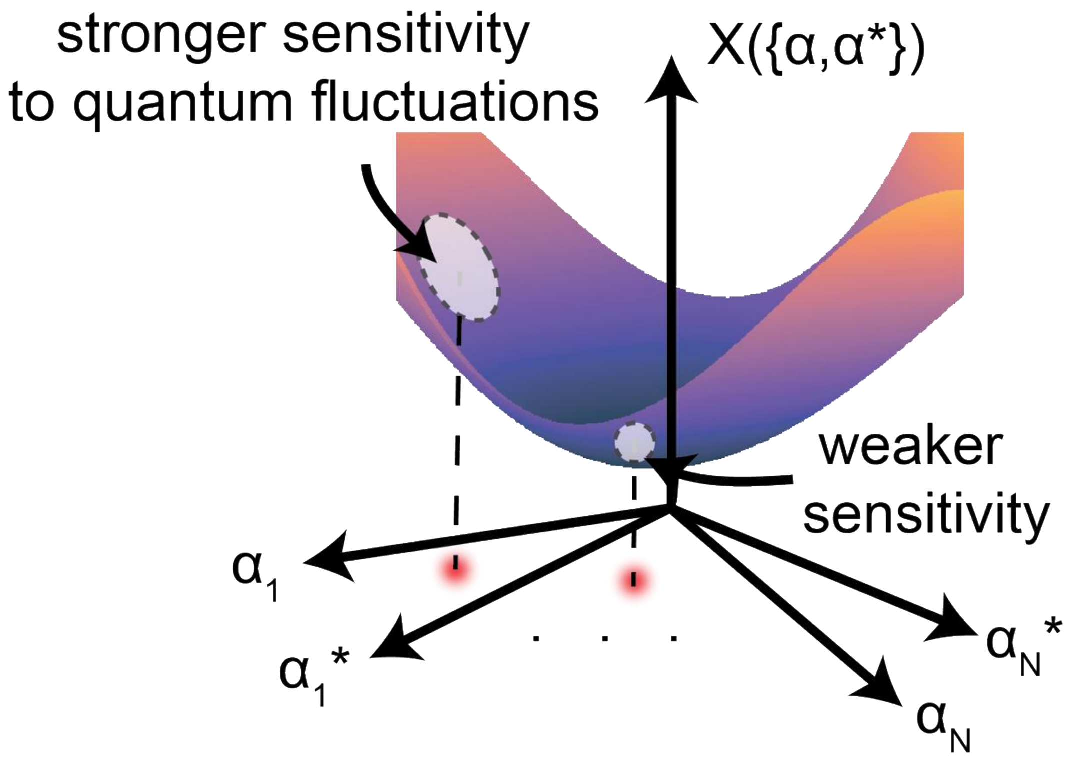
Quantum noise in nonlinear dynamical systems



Input-output picture of classical dynamics: $\alpha_i(t) = F[\{\alpha_j(0), \alpha_j^*(0)\}]$

Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Quantum sensitivity analysis of nonlinear dynamics

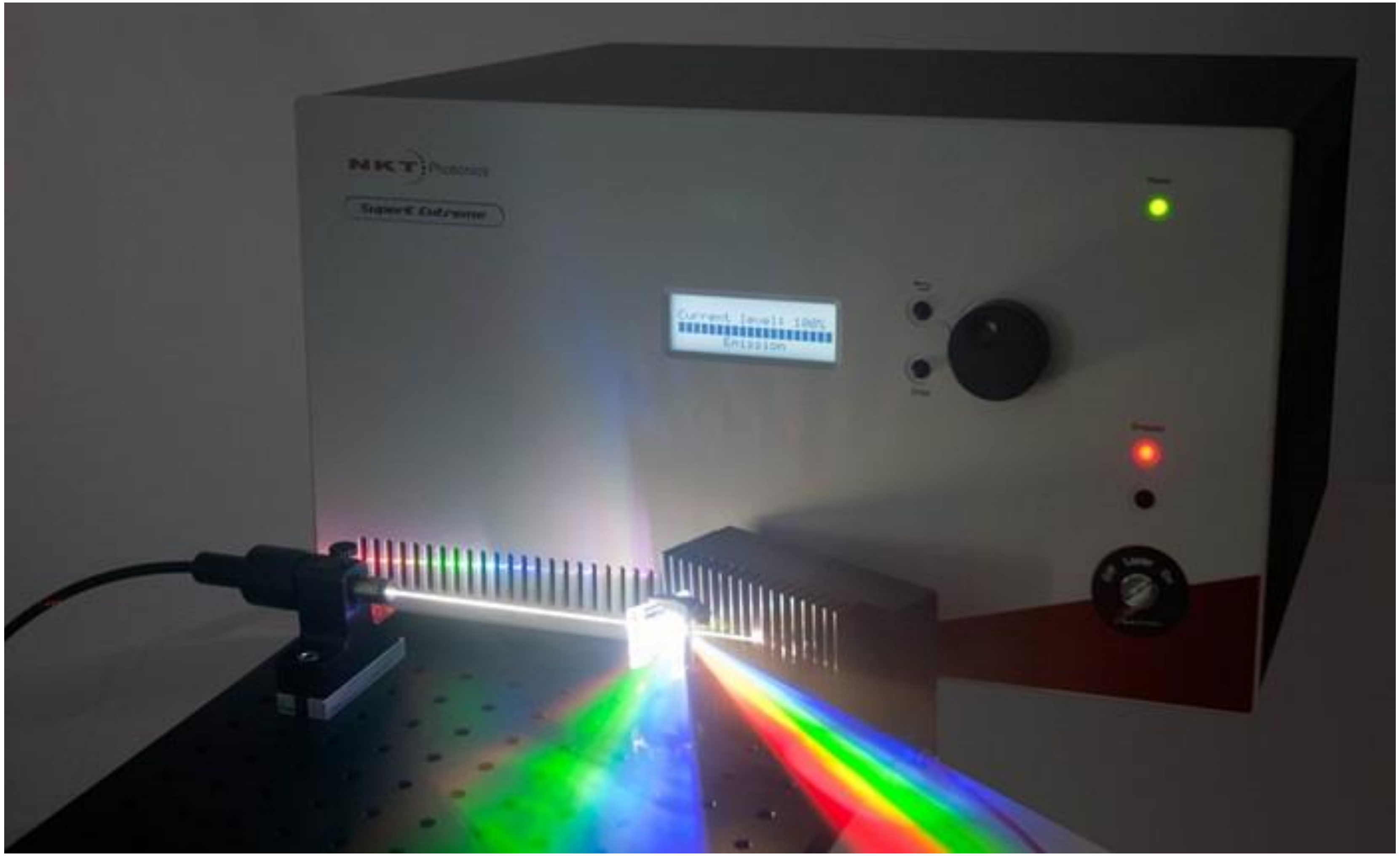


Crux: analyze response of observable to quantum or other noise in the initial conditions of the classical theory. Generally, noise of observable *X* given by:

$$(\Delta X(t))^2 = \begin{pmatrix} \frac{\partial X(t)}{\partial \boldsymbol{\alpha}(0)} & \frac{\partial X(t)}{\partial \boldsymbol{\alpha}^*(0)} \end{pmatrix}^T \begin{pmatrix} \langle \delta\boldsymbol{\alpha}\delta\boldsymbol{\alpha}\rangle & \langle \delta\boldsymbol{\alpha}\delta\boldsymbol{\alpha}^*\rangle \\ \langle \delta\boldsymbol{\alpha}^*\delta\boldsymbol{\alpha}\rangle & \langle \delta\boldsymbol{\alpha}^*\delta\boldsymbol{\alpha}^*\rangle \end{pmatrix} \begin{pmatrix} \frac{\partial X(t)}{\partial \boldsymbol{\alpha}(0)} & \frac{\partial X(t)}{\partial \boldsymbol{\alpha}^*(0)} \end{pmatrix}$$

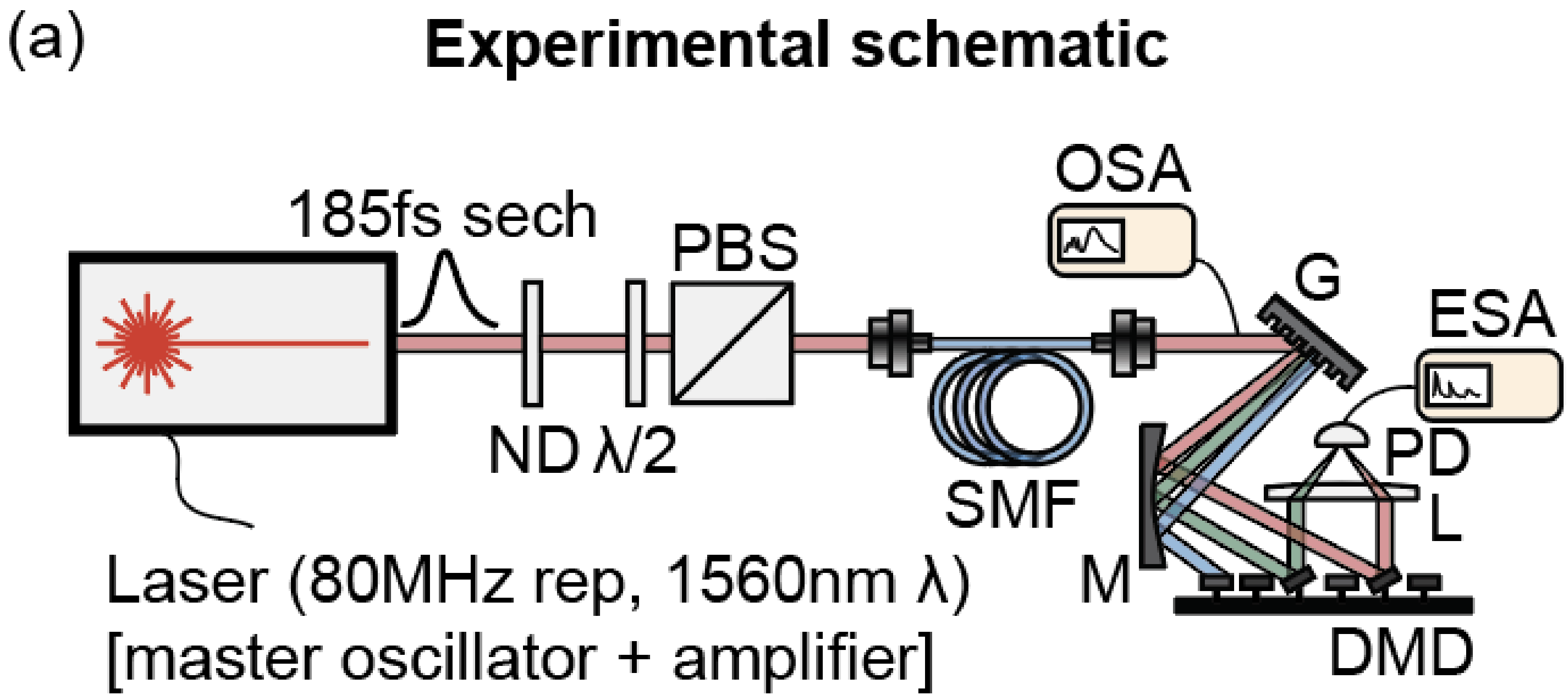
Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Quantum optics of supercontinuum generation



“White as a lamp, bright as a laser”

A new low noise regime in supercontinuum generation



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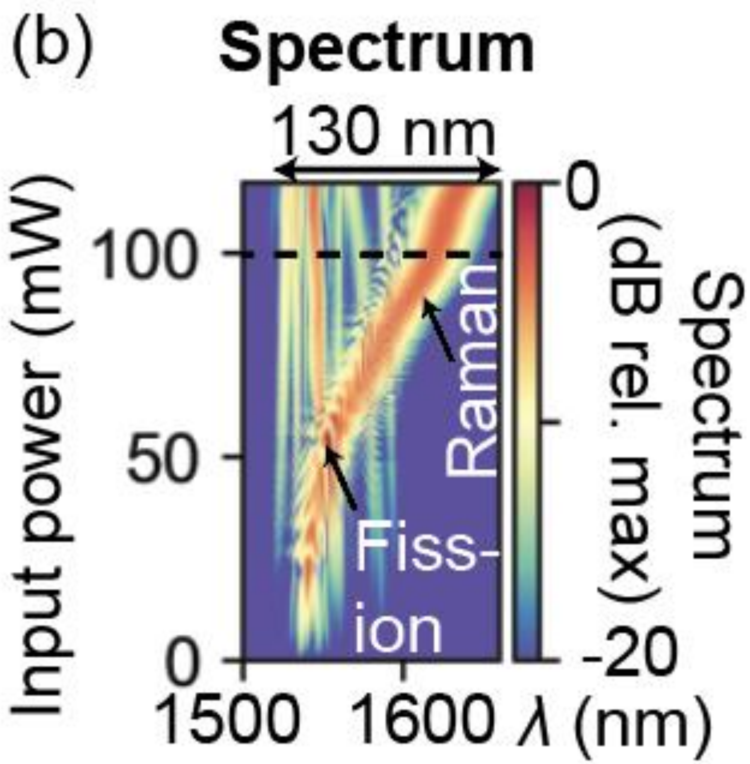

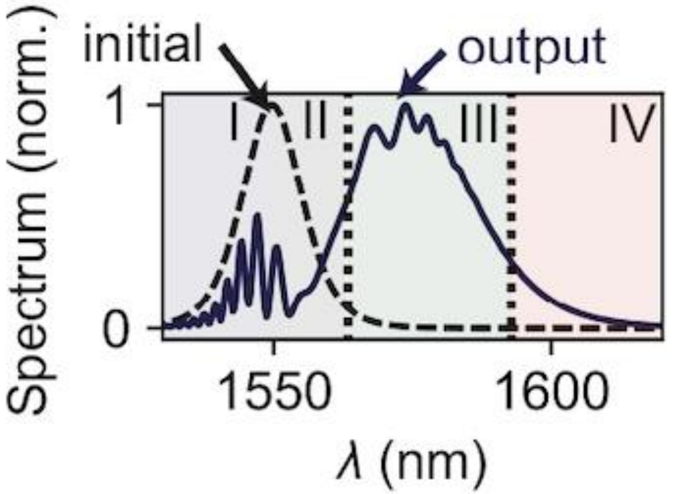

I: SPM + dispersive waves
II: SPM + Raman sol. (blue)
III: Raman sol. (peak)
IV: Raman sol. (red)

Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

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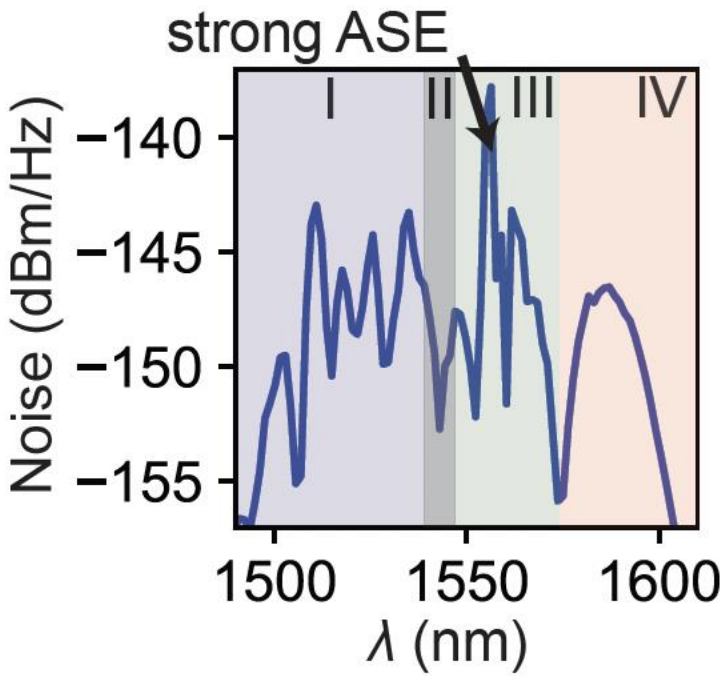

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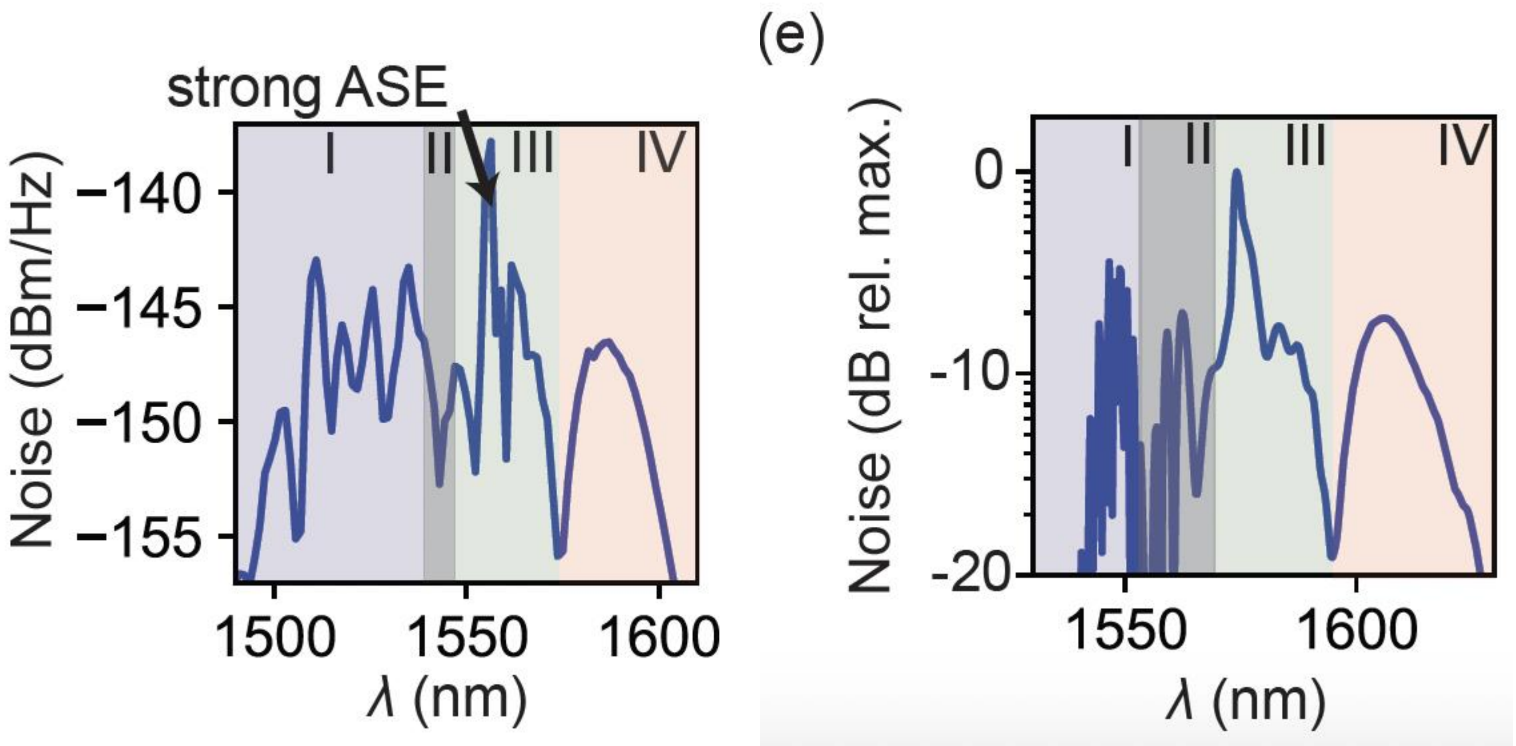

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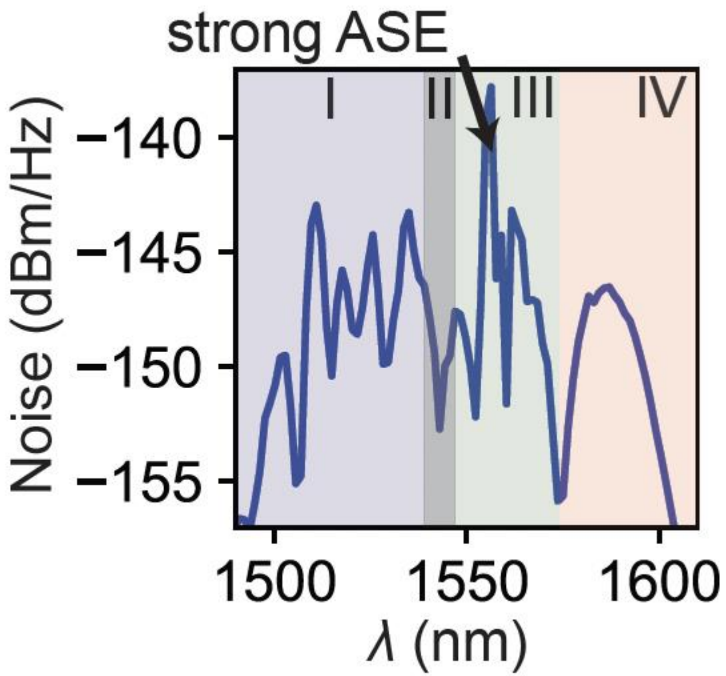

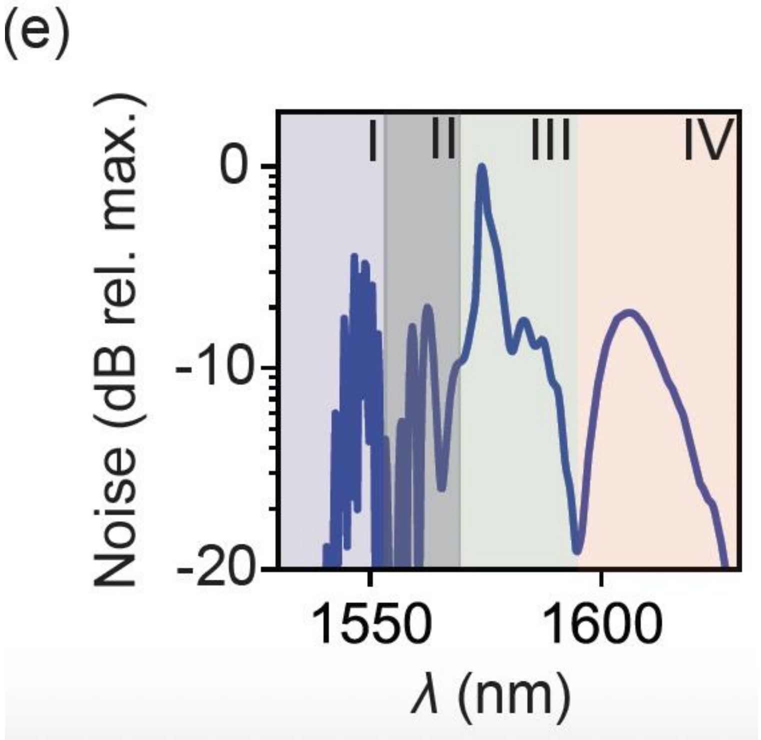

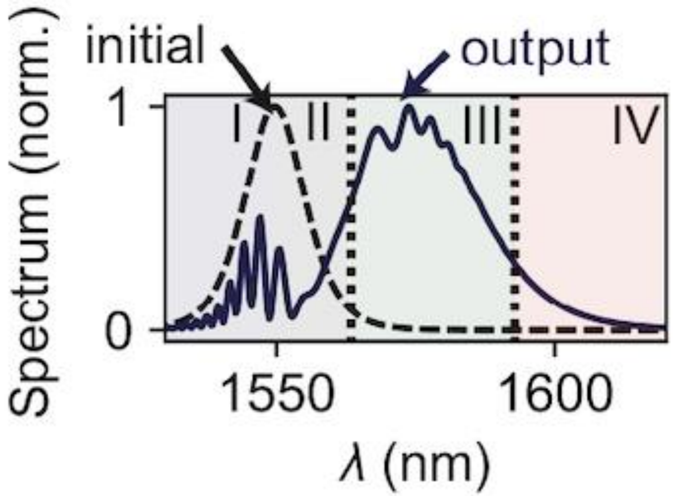

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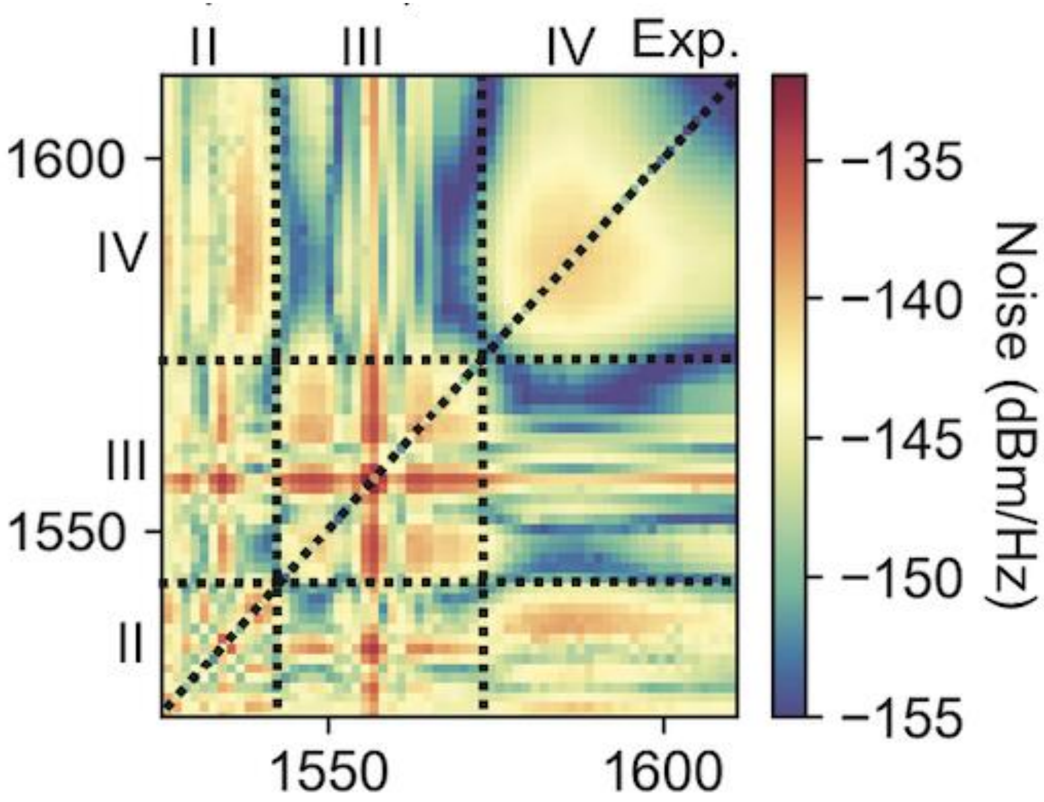
Low noise multi-wavelength states in supercontinuum



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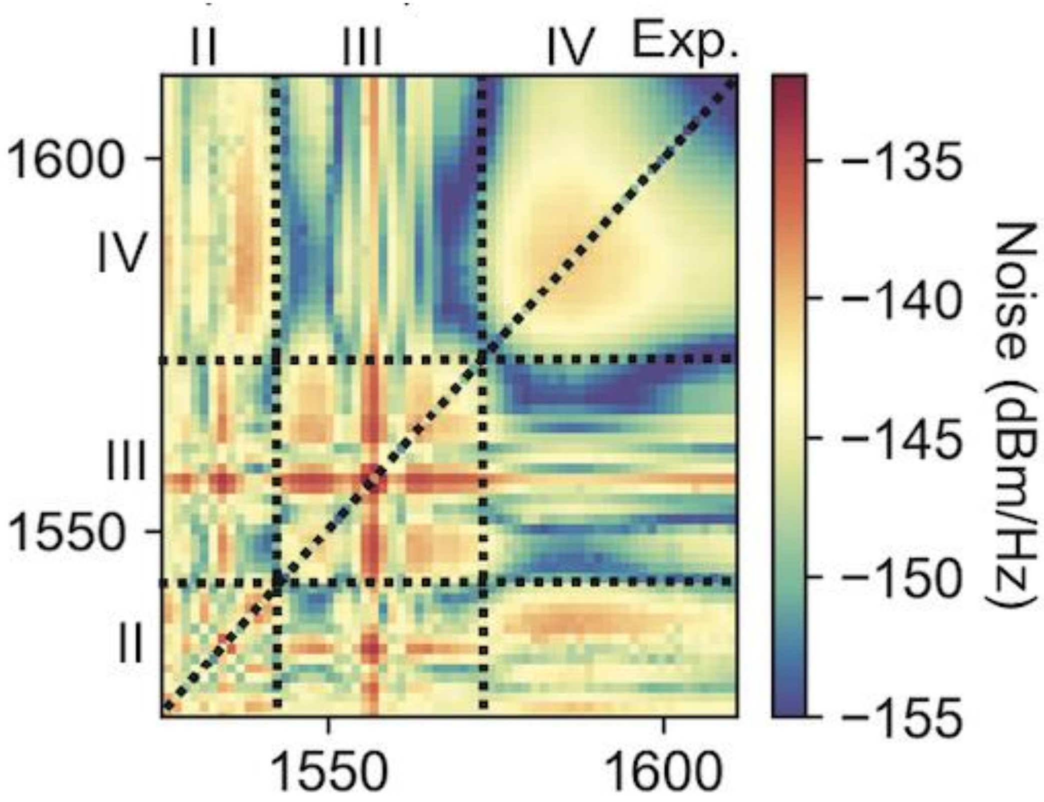

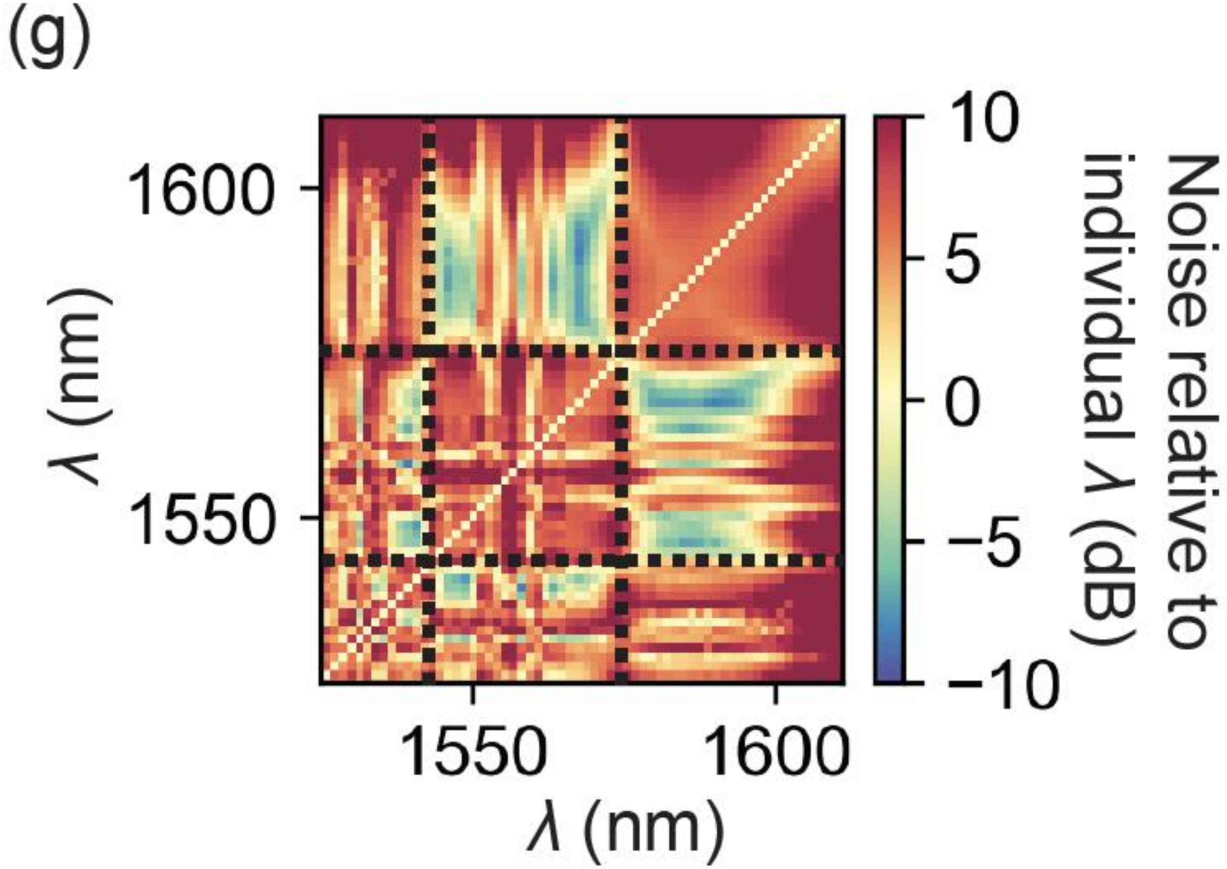

Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

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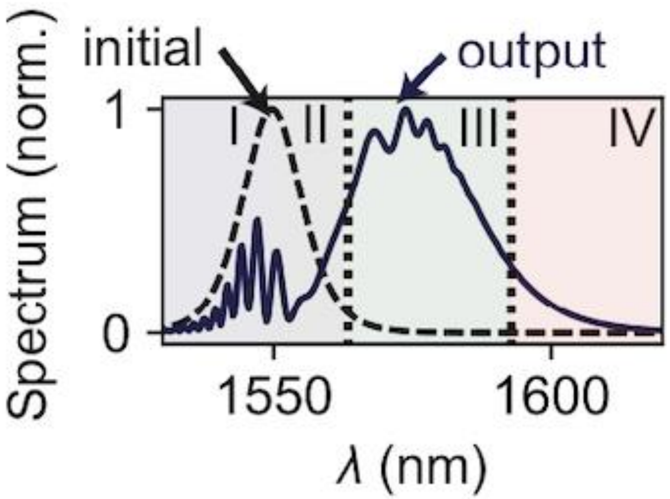

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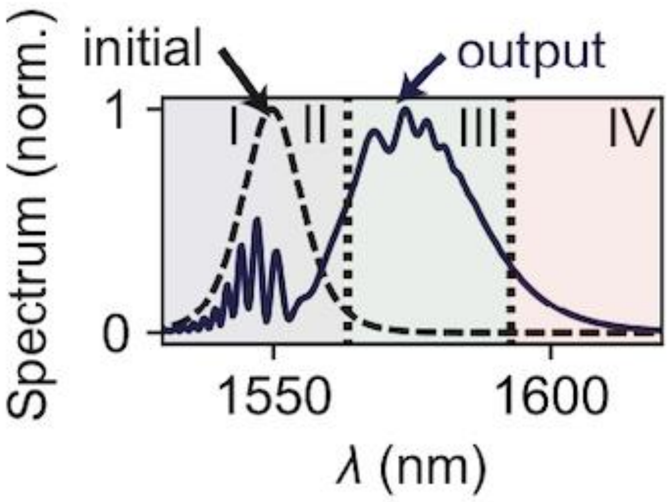

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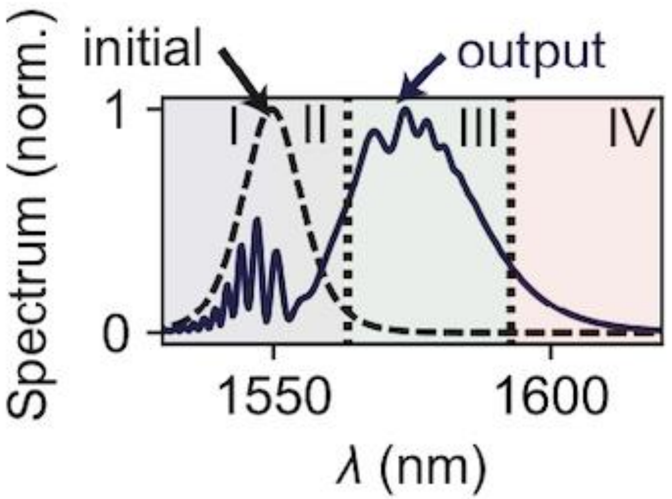
For the example of the low-noise band in supercontinuum, we analyze:

$$\Delta(n_1 + n_2)^2 = ||\nabla_{\boldsymbol{\alpha}(\mathbf{0})}[n_1(t) + n_2(t)]||^2$$

Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Low noise multi-wavelength states in supercontinuum



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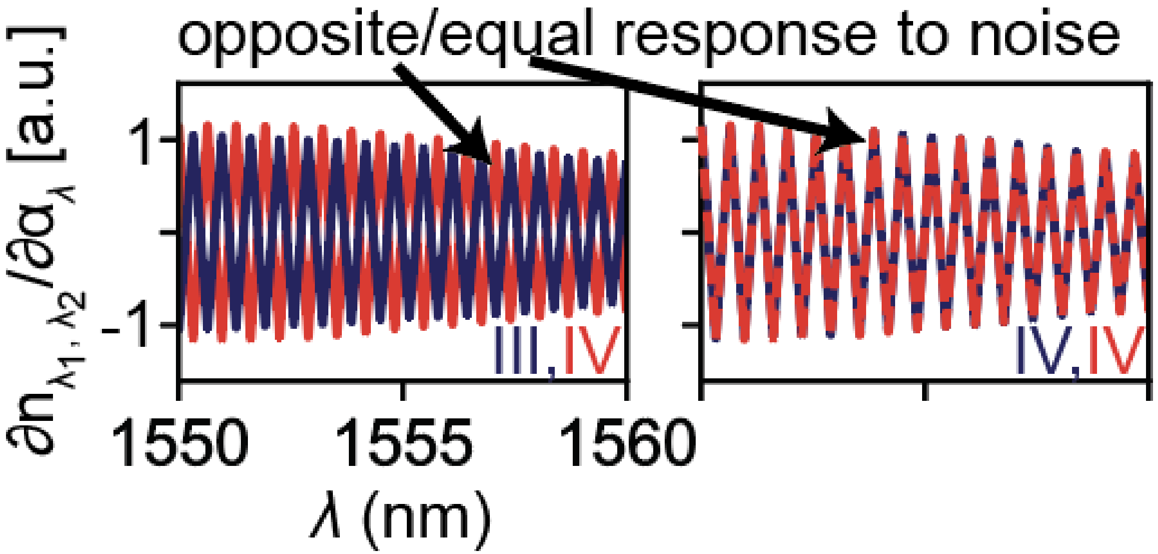
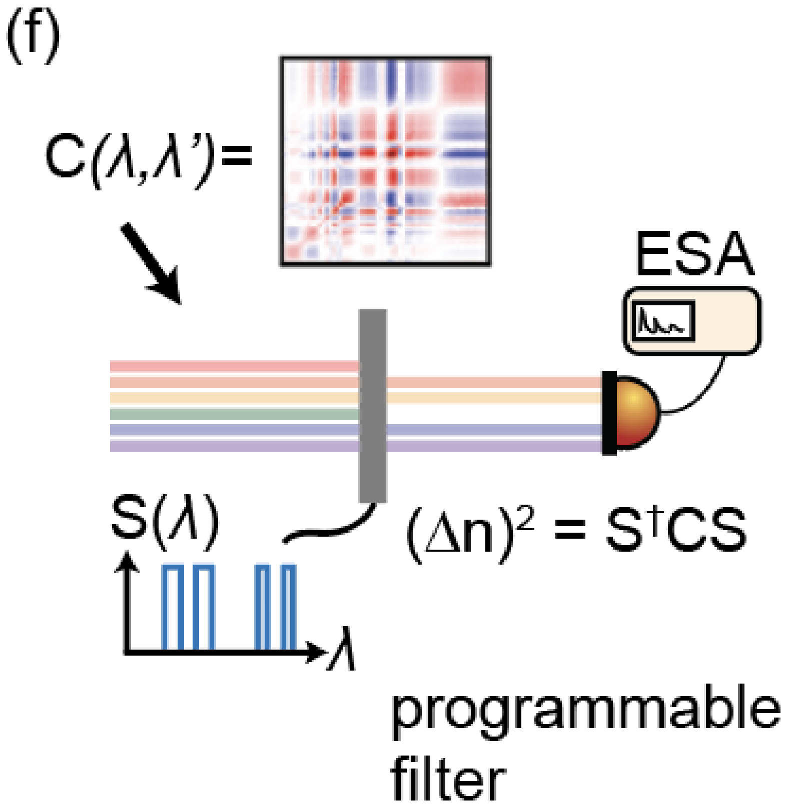


Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Squeezed states in supercontinuum



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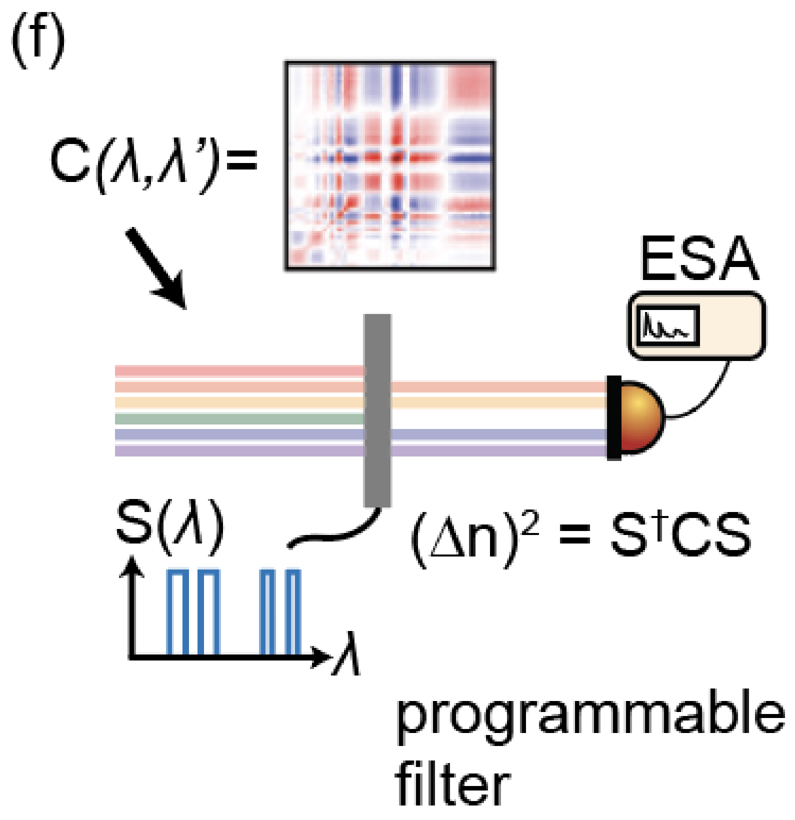

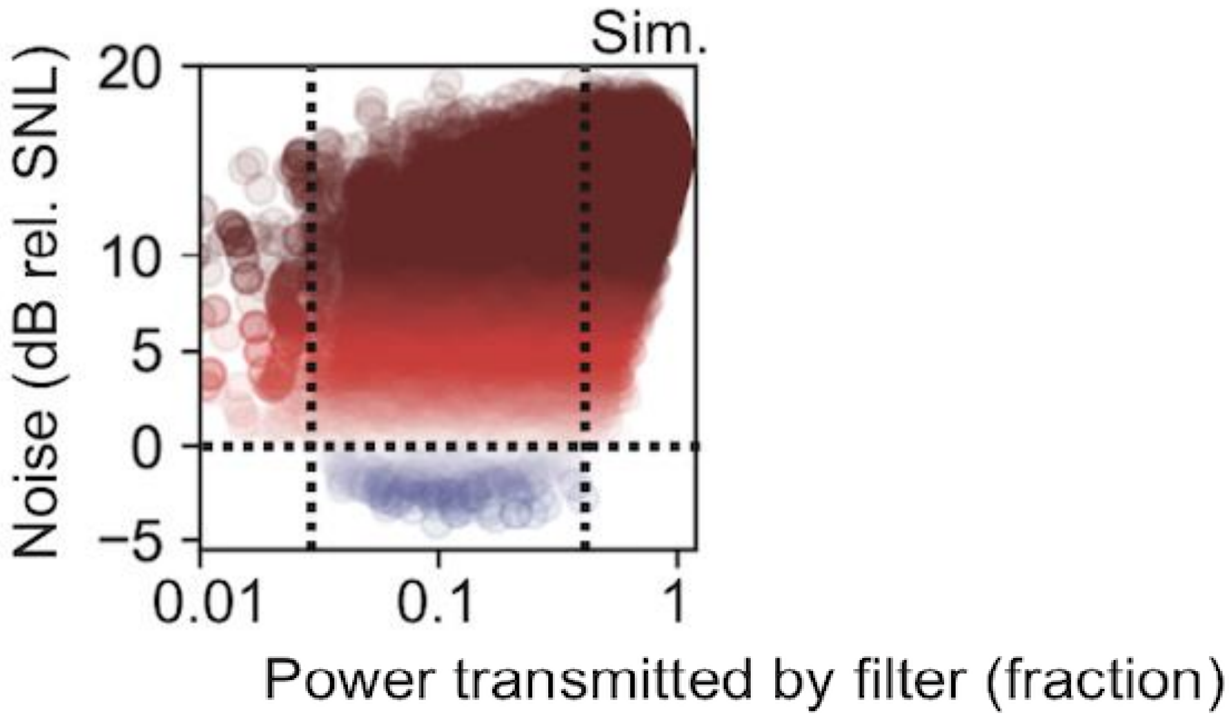

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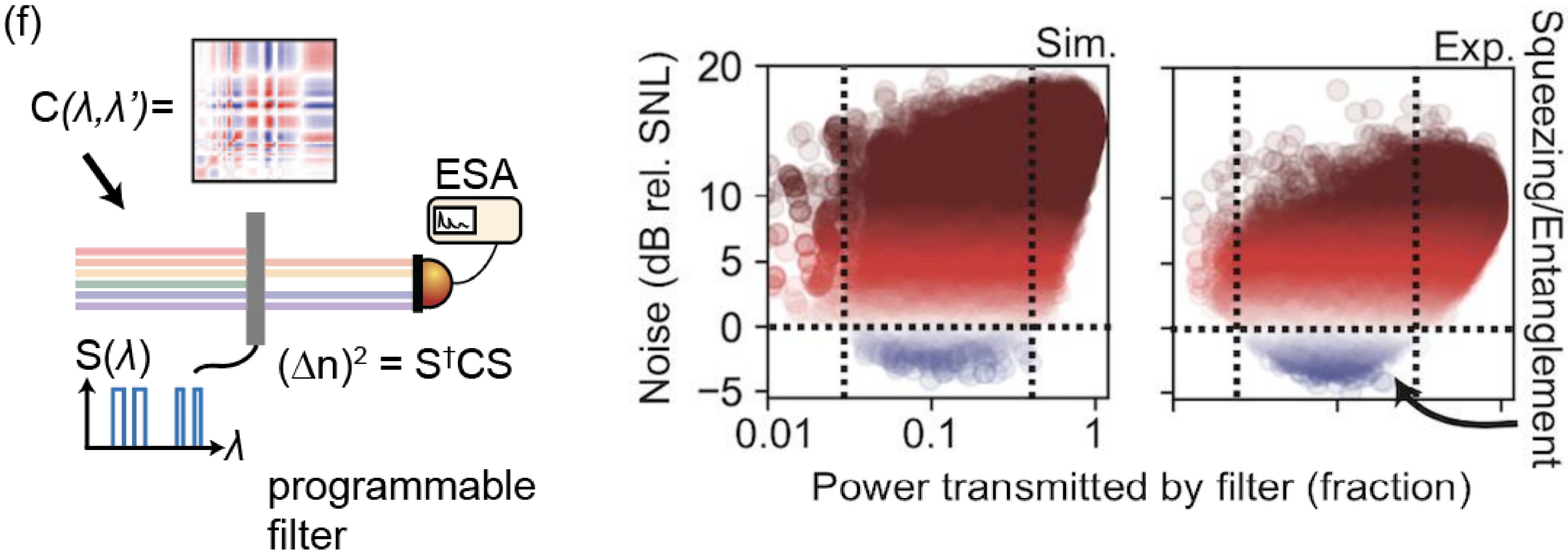
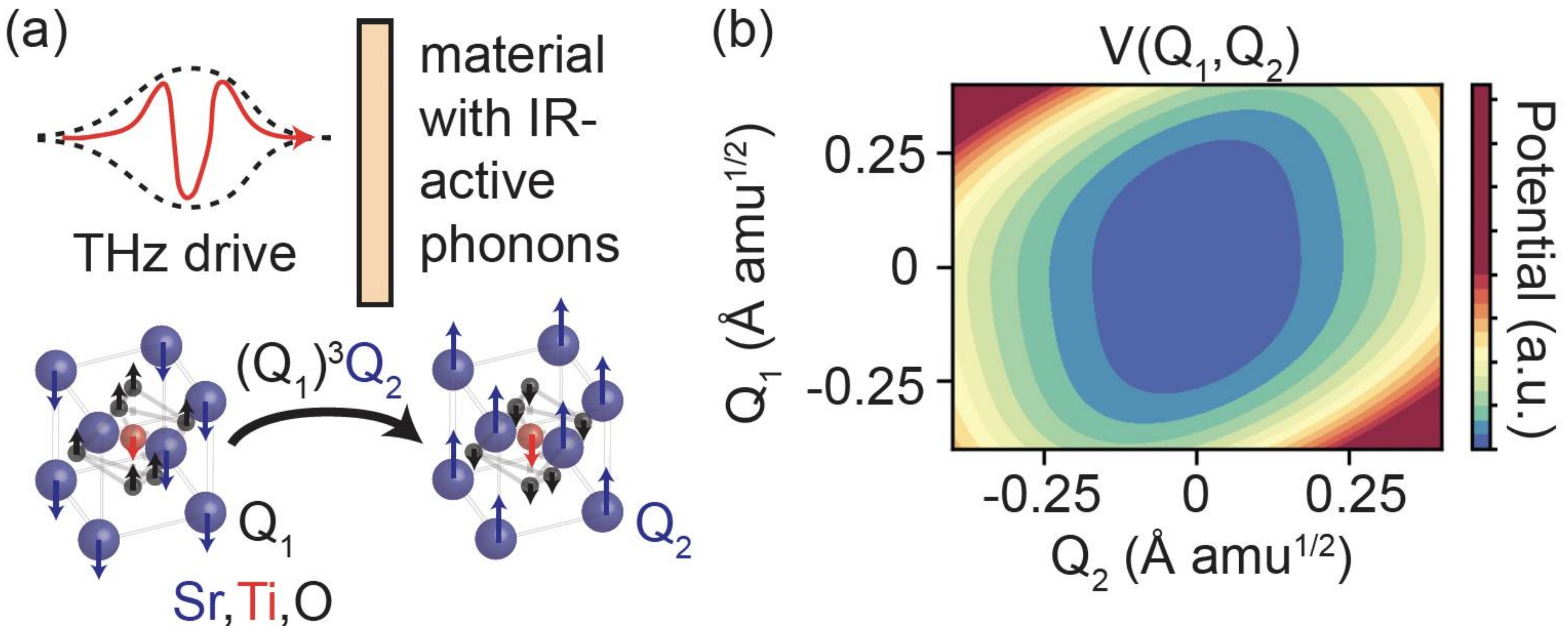


Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

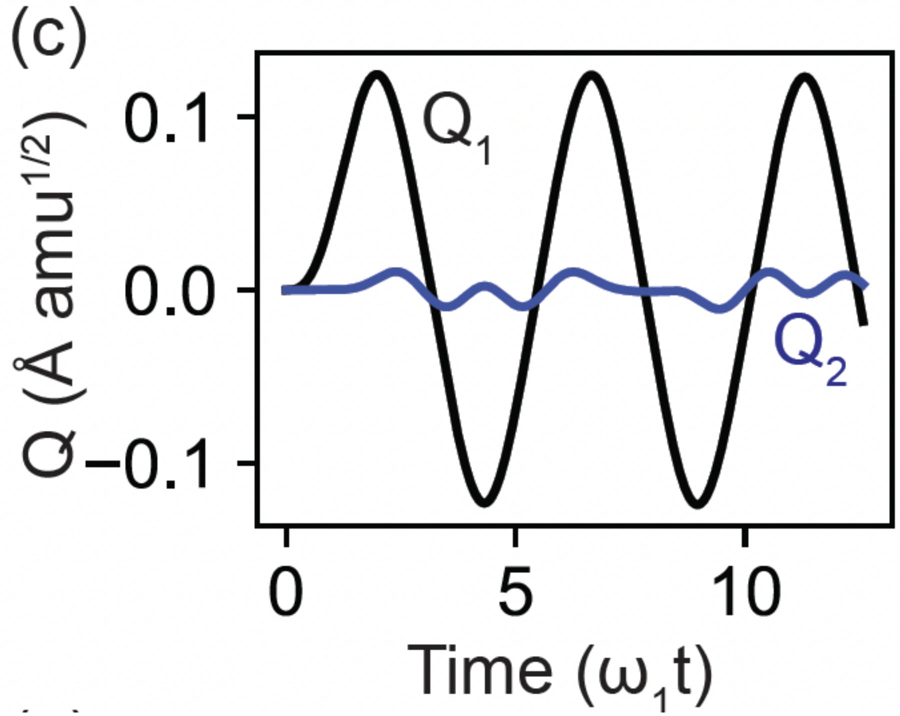
New directions with this theory

Phonon squeezing

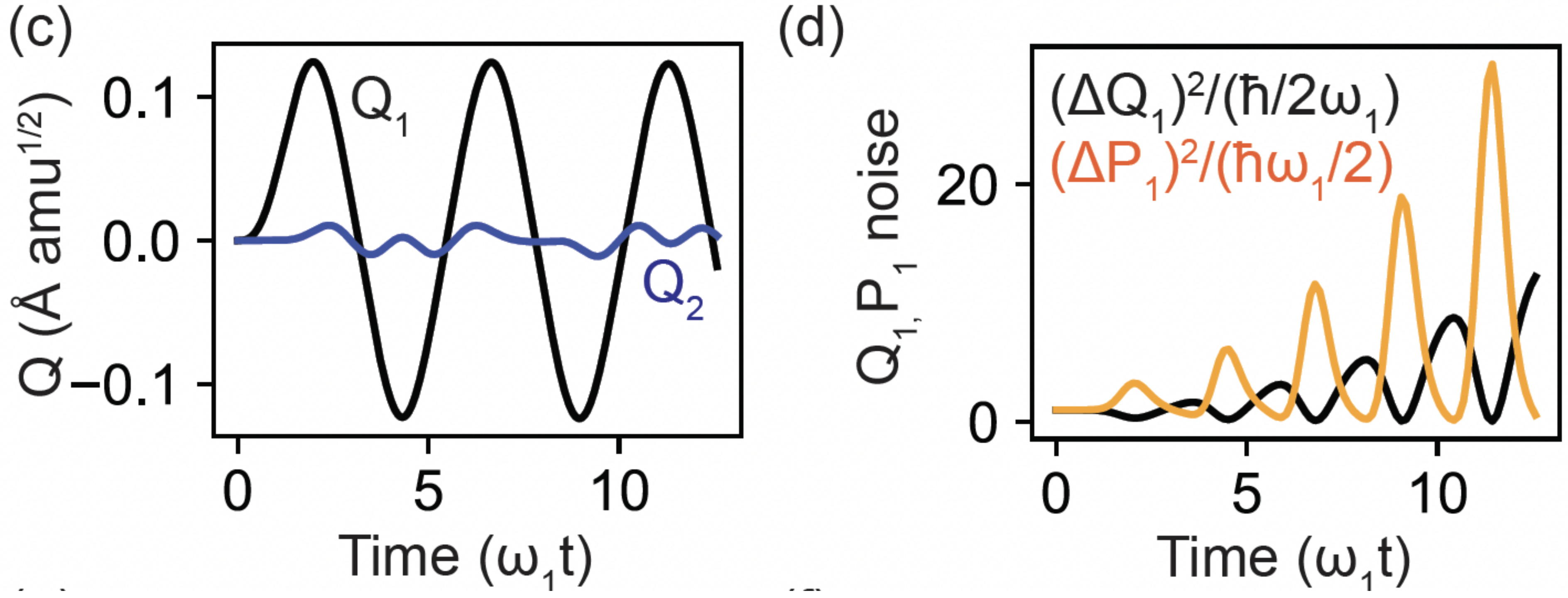


$$V = \frac{1}{2}\omega_1^2 Q_1^2 + \frac{1}{2}\omega_2^2 Q_2^2 + \frac{1}{4}\kappa_1 Q_1^4 + \frac{1}{4}\kappa_2 Q_2^4 + \chi Q_1^2 Q_2^2 + \psi_{12} Q_1^3 Q_2 + \psi_{21} Q_1 Q_2^3$$

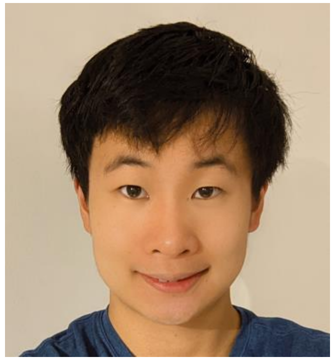
Phonon squeezing



Phonon squeezing



Jamison Sloan



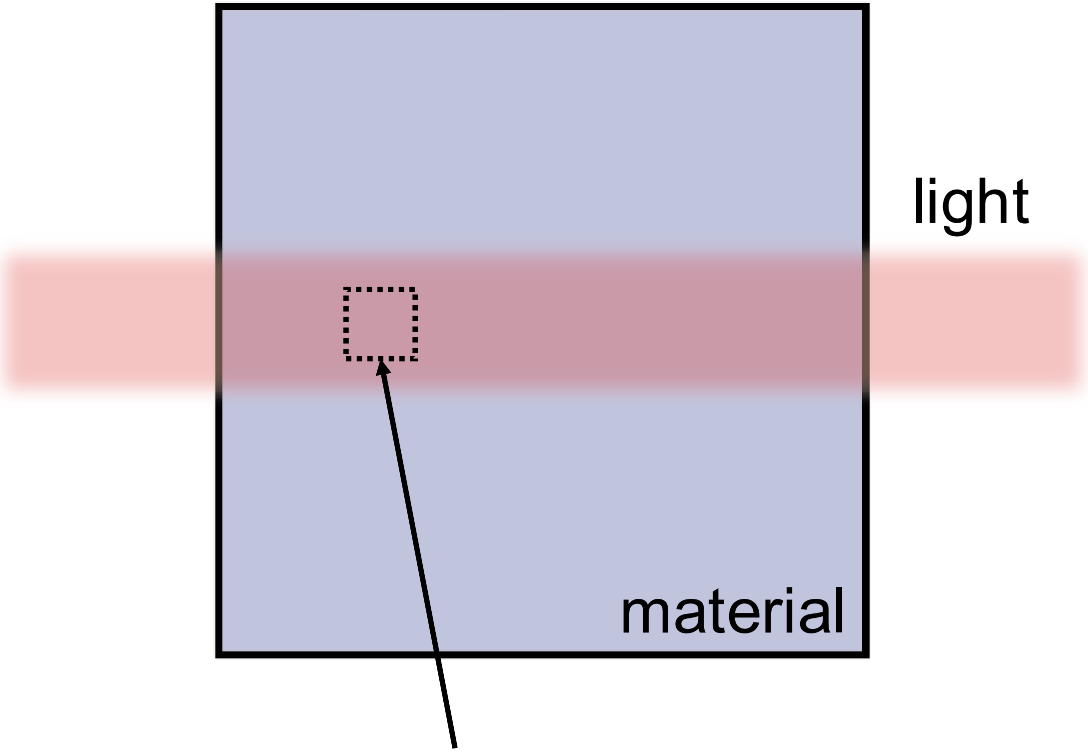
Sean Chen
(UROP)

Linh Nguyen
(UROP)

Generating fundamental quantum states of light in optics using nonlinear nanophotonics

Based on: **Rivera** *et al.* PNAS (2023), Chen and **Rivera** *et al.* arXiv:2312.07386 (2023)

Changing optical properties with one or few photons



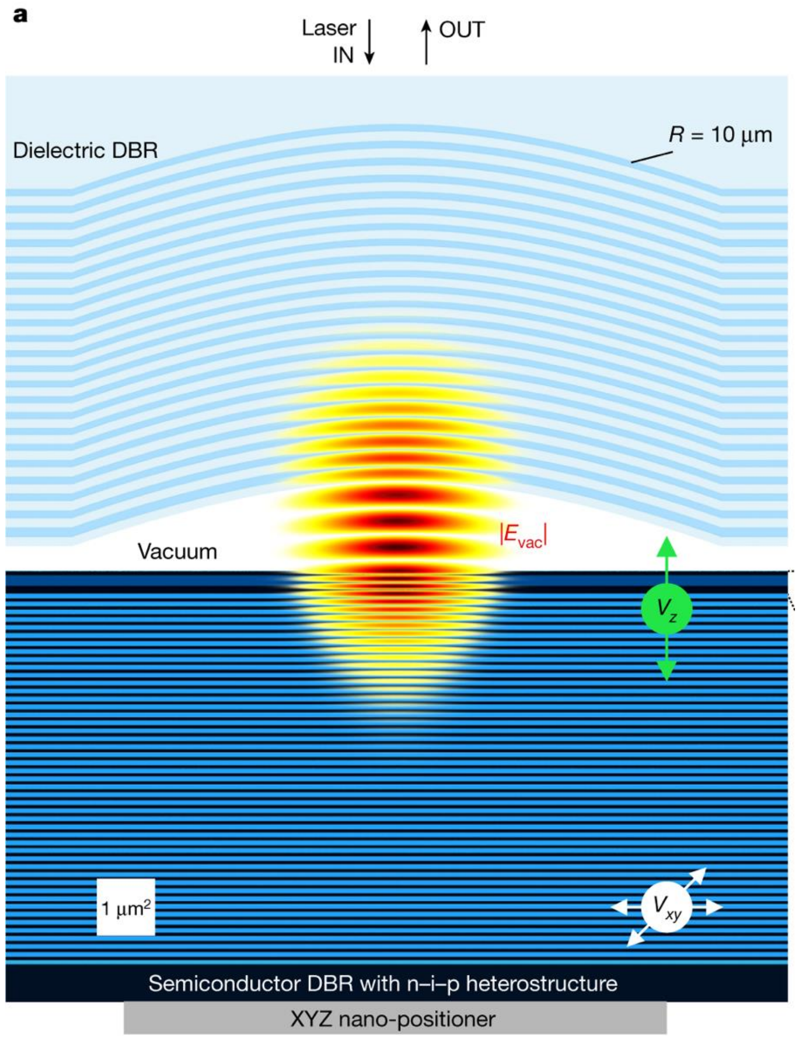
$$P_i = \epsilon_0 \left(\chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \dots \right)$$

Material polarization | Weak fields | Stronger driving fields (e.g., using focused lasers)

Relevant for "field of a single photon"? $E_{\mathrm{SP}} \sim \sqrt{\hbar\omega / 2\epsilon_0 V}$

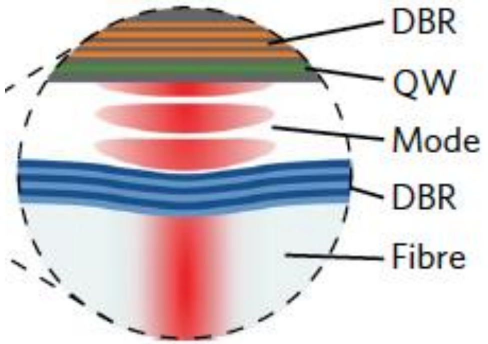
Systems for realizing nonlinear *optics* at few-photon scales

Two-level systems in cavities



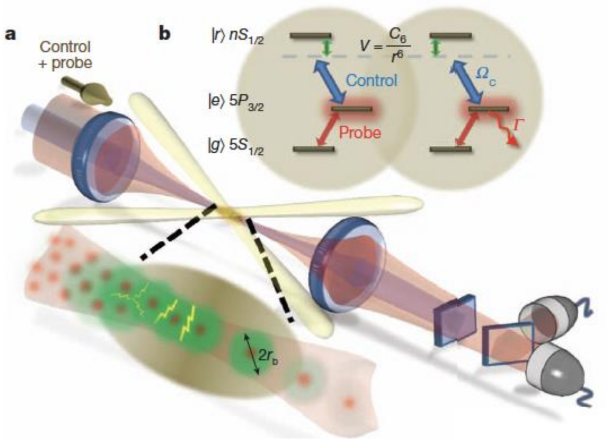
Najer *et al. Nature* (2019).

Exciton polaritons



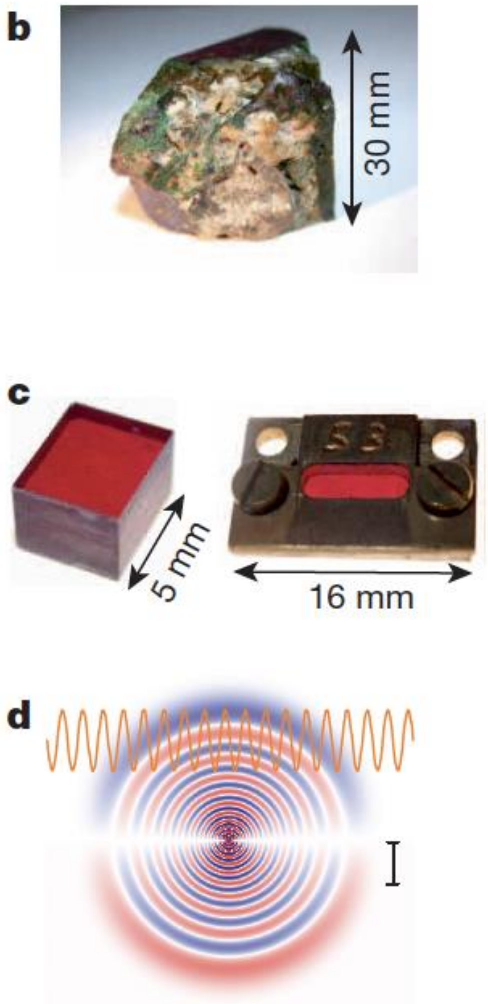
Delteil *et al* & Matutano-Munoz *et al. Nature Materials* (2019).

Cold atoms



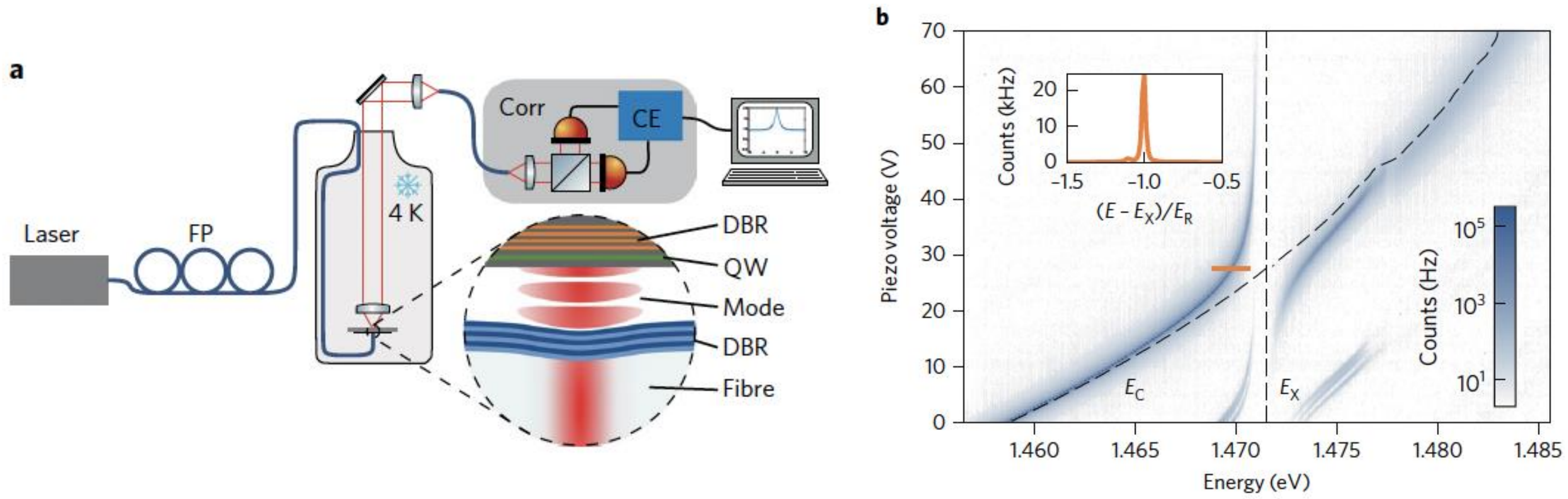
Peyronel *et al. Nature* (2013).

Rydberg excitons in Cu_2O



Kazimierczuk *et al. Nature* (2014).

Extremely strong Kerr nonlinearities from semiconductors (excitons) in microcavities



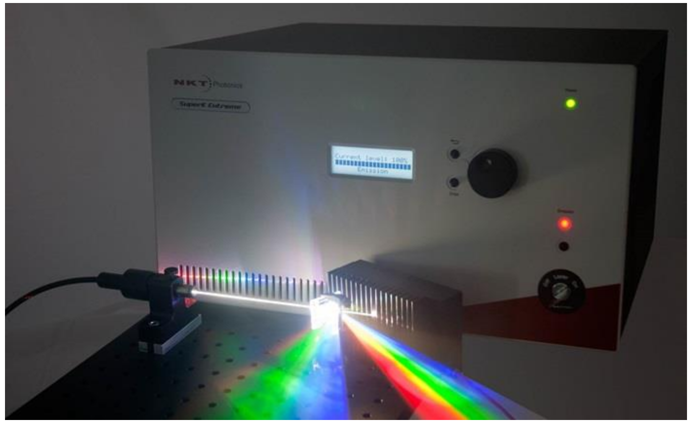
Quantum Kerr physics in exciton-polaritons
Imamoglu *et al.* Physical Review Letters (1997).
Bramati *et al.* Nature Communications (2014).
Fink *et al.* Nature Physics (2018).
Delteil & Fink *et al.* Nature Materials (2019).

How close are the implementations?

- Kerr nonlinearity within factor of 10 of loss (single-photon nonlinear regime)
- Exciton-polaritons even recently interfaced with gratings with BICs (Ardizzone *et al.* Nature (2022)).

Why? Beyond "Gaussian" states of light

Macroscopic nonlinear optics



Quantum state transformation

$$a_i^{\text{out}} = \alpha_i + \sum_j \mu_{ij} \delta a_j^{\text{in}} + \nu_{ij} (\delta a_j^{\text{in}})^\dagger$$

Few-photon nonlinear optics

Photonic Fock states

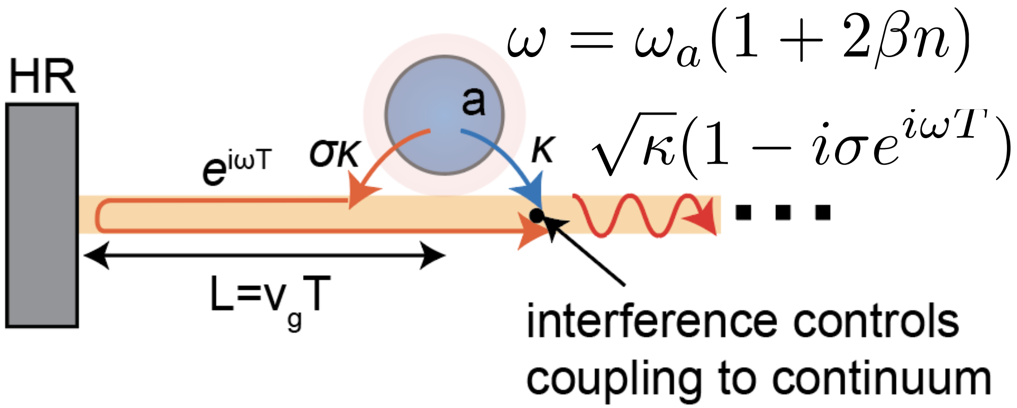
$$|\psi\rangle \sim (a^\dagger)^n |0\rangle$$

Schrodinger "cat" states

$$|\psi\rangle \sim |\alpha\rangle \pm |-\alpha\rangle$$

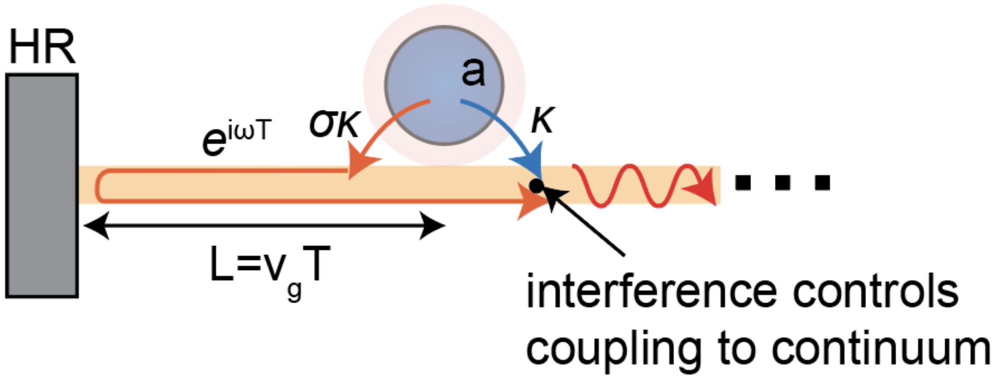
These and many other states have not been deterministically prepared in optics!

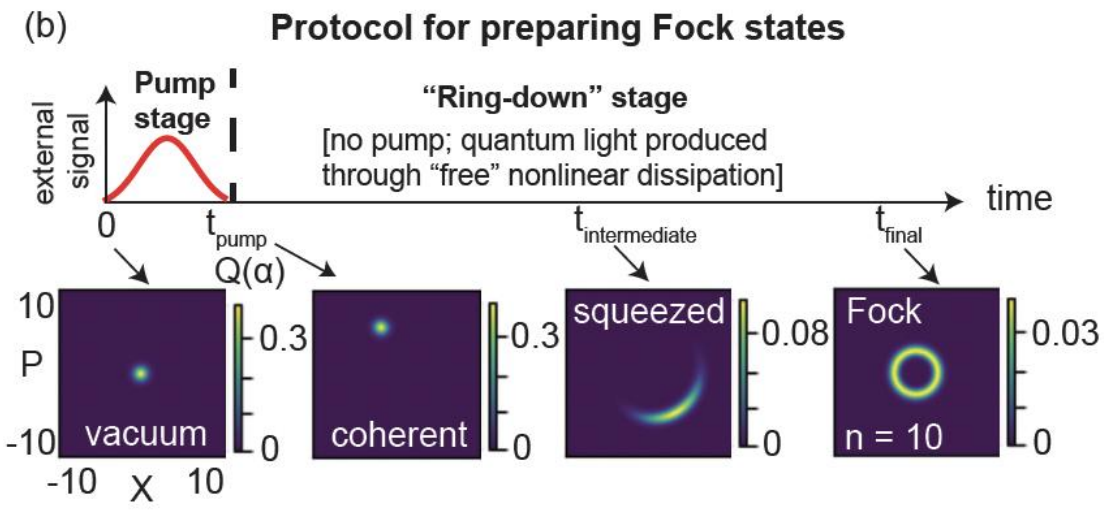
Photonic structuring + nonlinearity: n-photon bound states



Rivera*, Sloan* *et al.* PNAS (2023).

Deterministic generation of large optical Fock states





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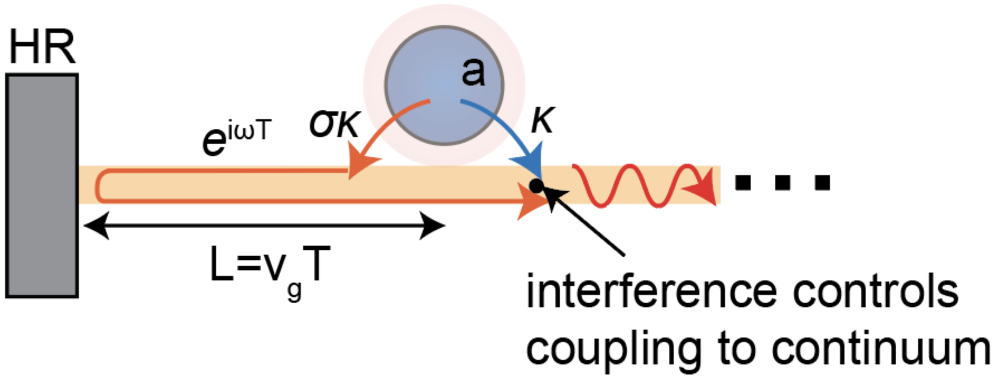

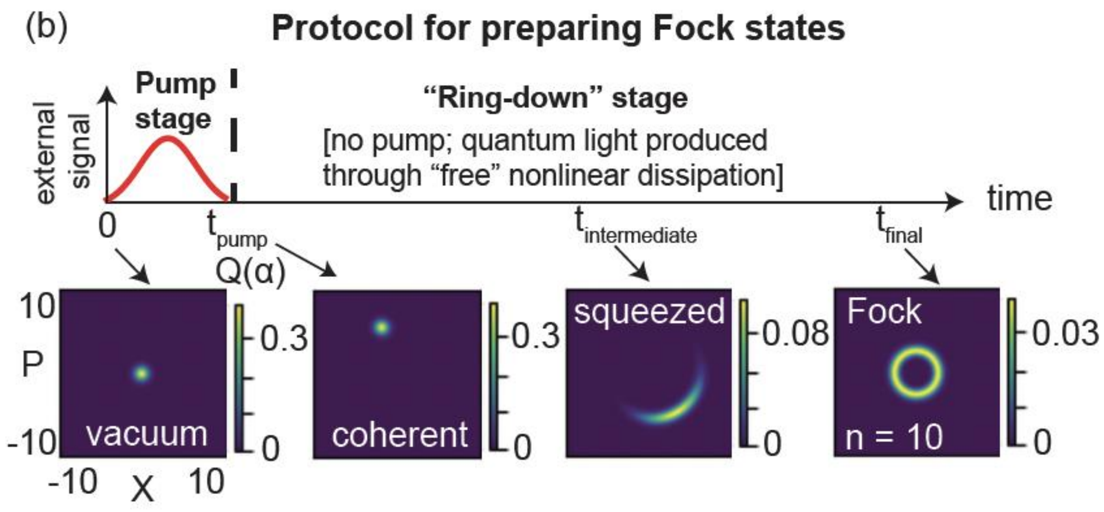

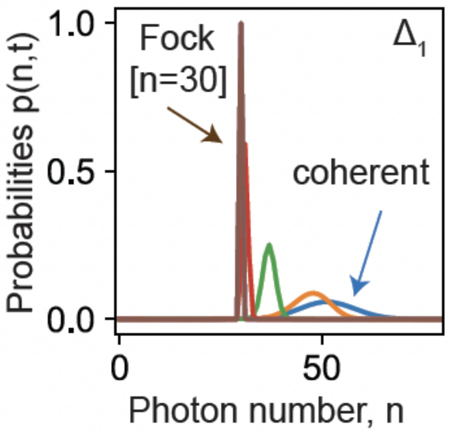

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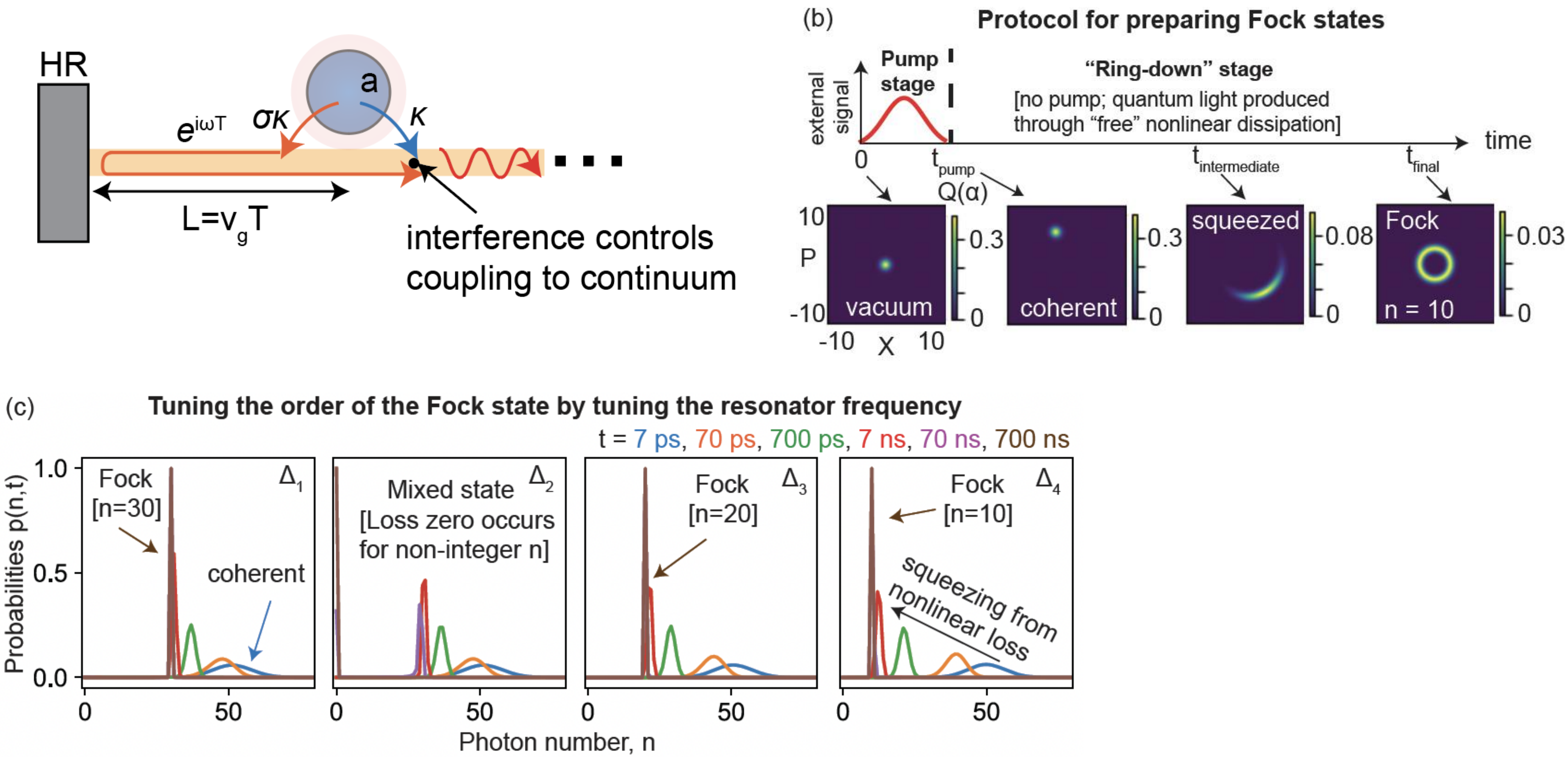
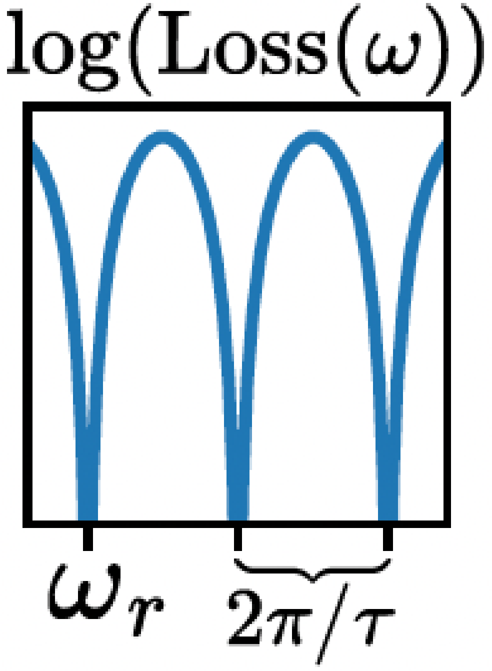


See also: Nguyen, Sloan, **Rivera** *et al.* PRL (2023) [low-noise lasers], Pontula*, Sloan*, **Rivera,** *et al.* arXiv:2212.07300 [THz squeezing], Sloan*, **Rivera*** *et al.* arXiv:2309.09863 [cw stabilization of Fock states]

Controlling photonic loss: Schrodinger cat states



Chen, **Rivera** *et al.* arXiv:2312.07386 (2023).

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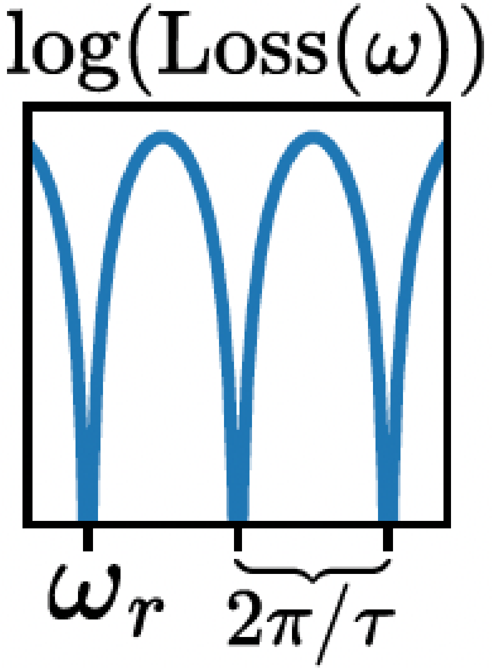
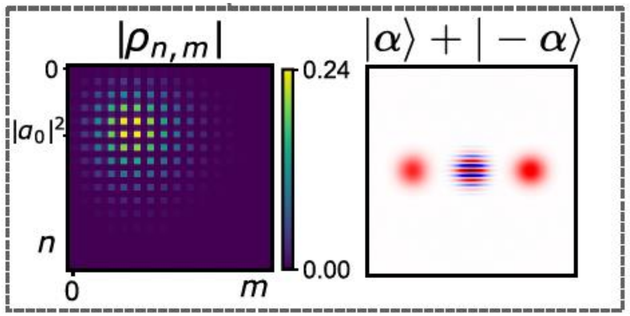
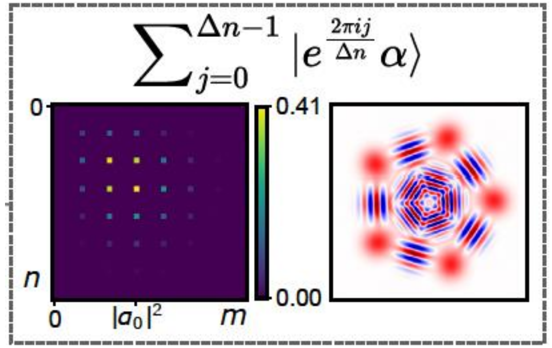


Zero loss every
2 photons

Zero loss every
5 photons

2-legged Schrodinger cat





5-legged Schrodinger cat

Chen, **Rivera** *et al.* arXiv:2312.07386 (2023).

The future: quantum and nonlinear optics in new directions

Quantum optics in emerging material systems

Goal: understanding quantum light interactions with material degrees of freedom, where material degrees of freedom strongly interact. Using this to develop nonlinear optics in the ”mesoscopic” regime of more than 1 or 2 photons.

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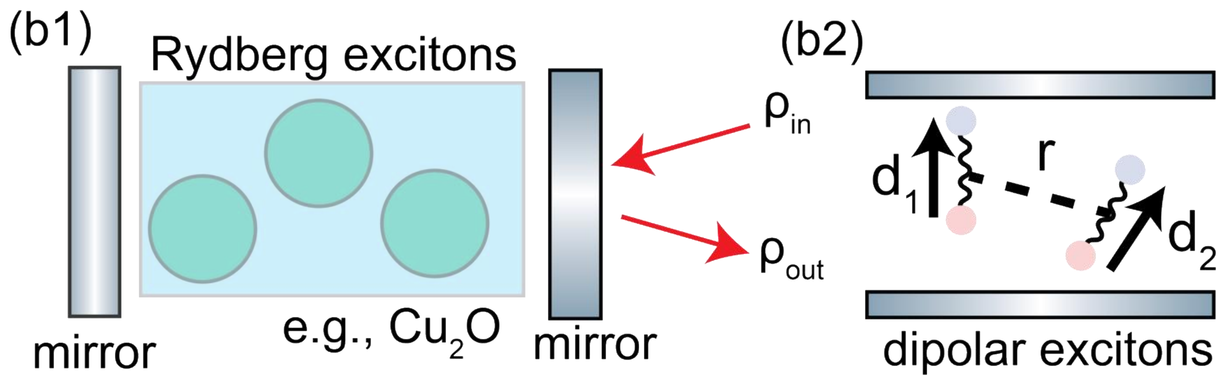
Excitonic systems: single-photon nonlinearities



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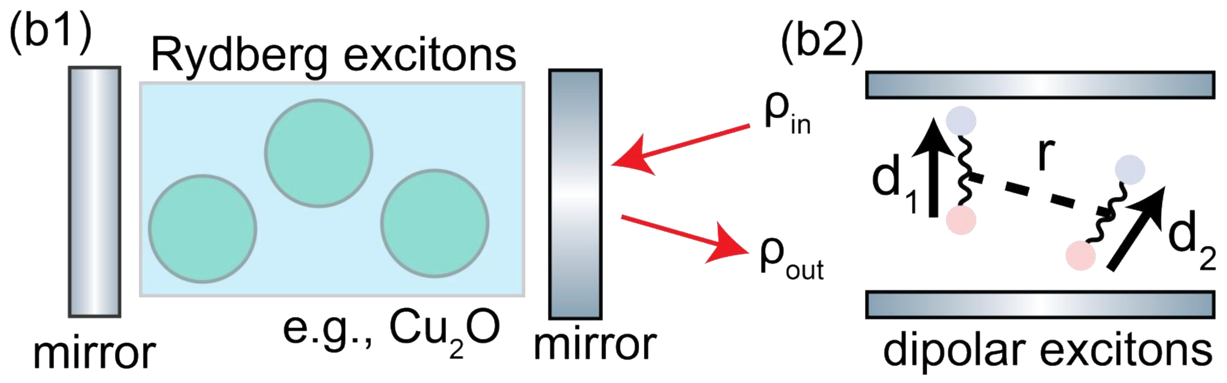
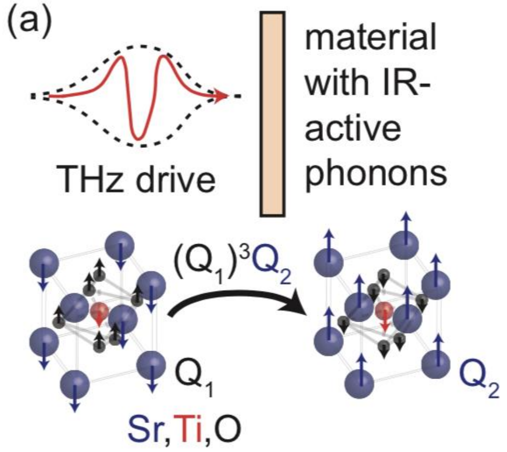


Phononic systems: quantum optics of lattice vibrations



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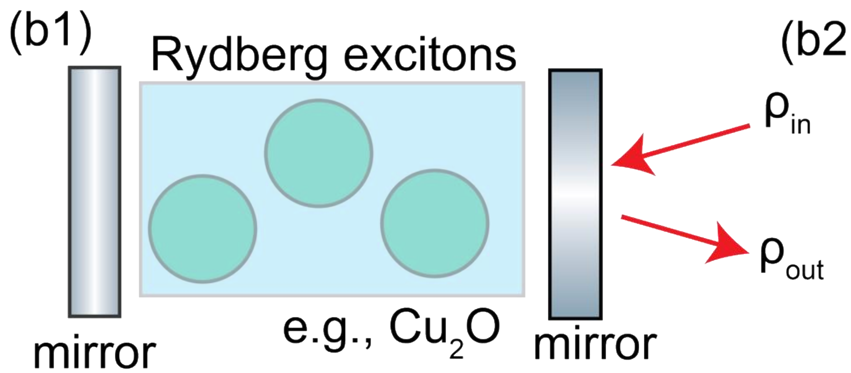
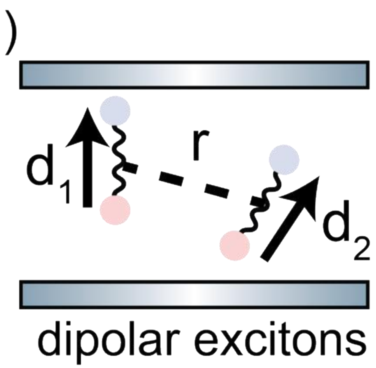
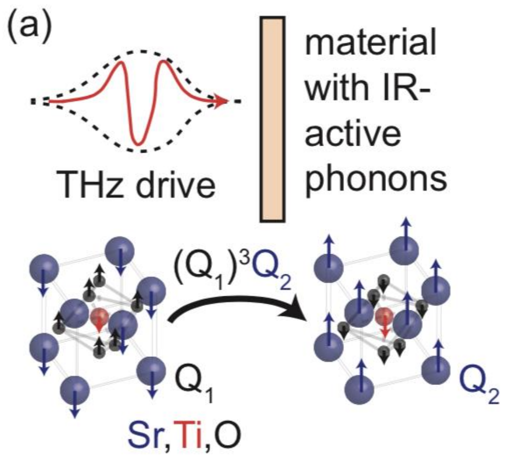


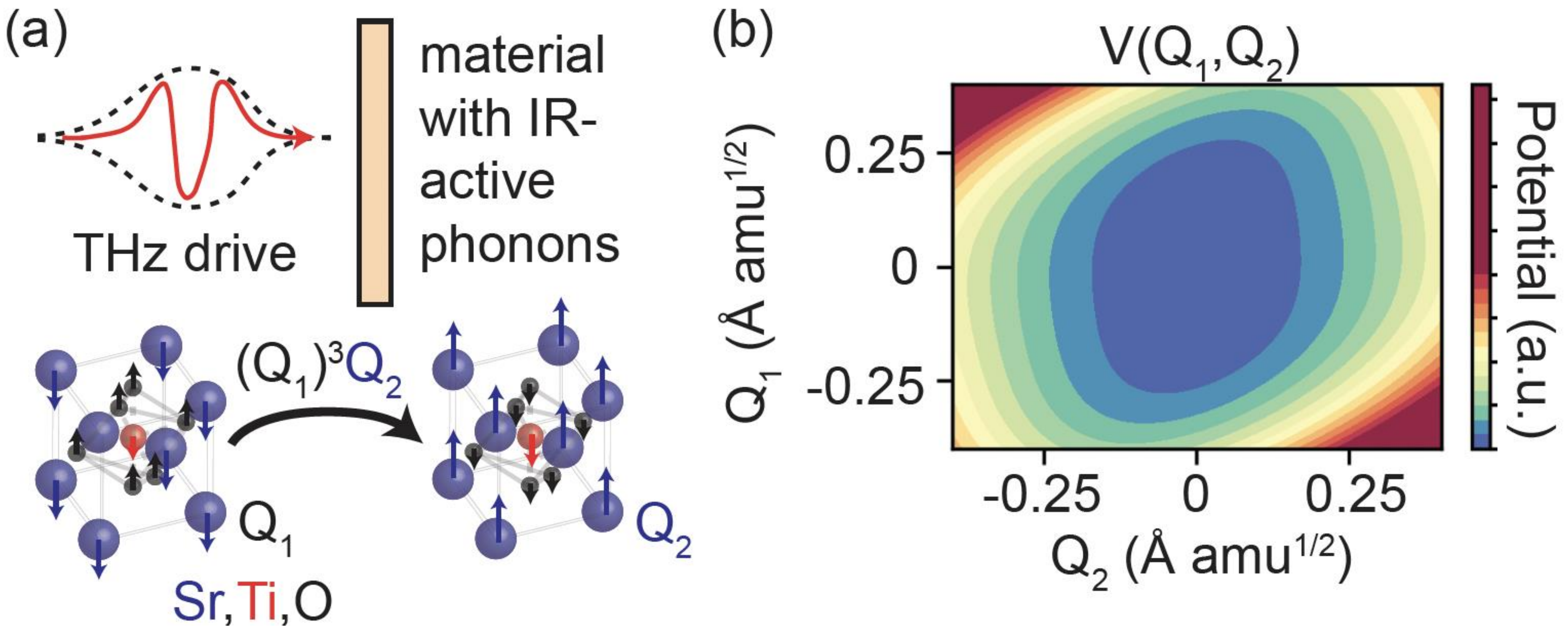


Phononic systems: quantum optics of lattice vibrations



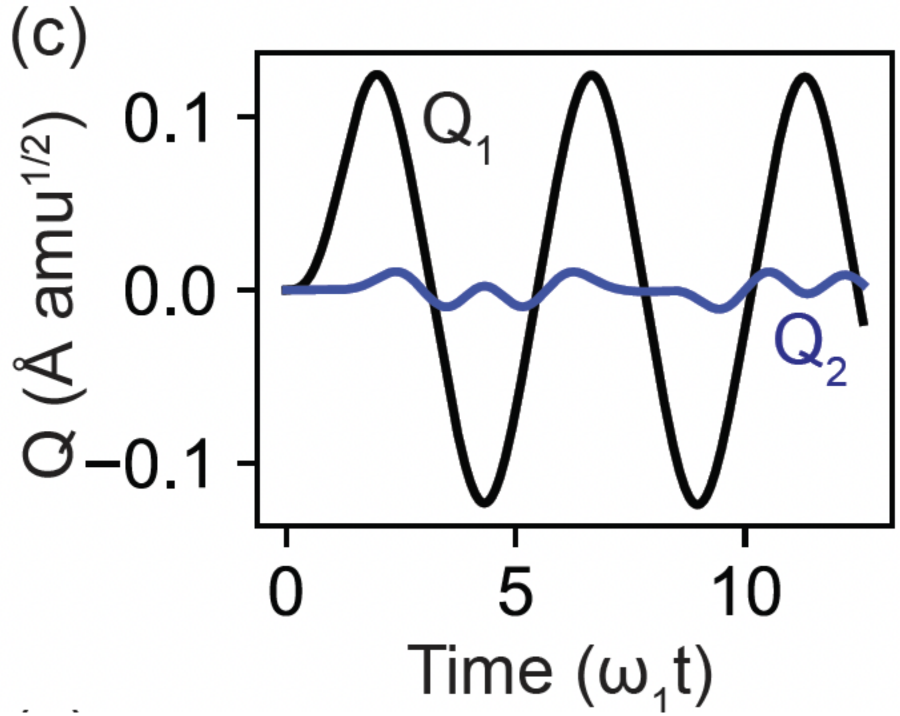
Then what? Excitons – understanding few-photon nonlinear response allows us to build architectures to control. Phonons – lets us develop quantum light at new frequencies (THz), and may lead to new concepts for controlling material phases

Phonon squeezing



$$
\begin{aligned}
V &= \tfrac{1}{2}\omega_1^2 Q_1^2 + \tfrac{1}{2}\omega_2^2 Q_2^2 + \tfrac{1}{4}\kappa_1 Q_1^4 + \tfrac{1}{4}\kappa_2 Q_2^4 \\
&\quad + \chi Q_1^2 Q_2^2 + \psi_{12} Q_1^3 Q_2 + \psi_{21} Q_1 Q_2^3
\end{aligned}
$$

Phonon squeezing



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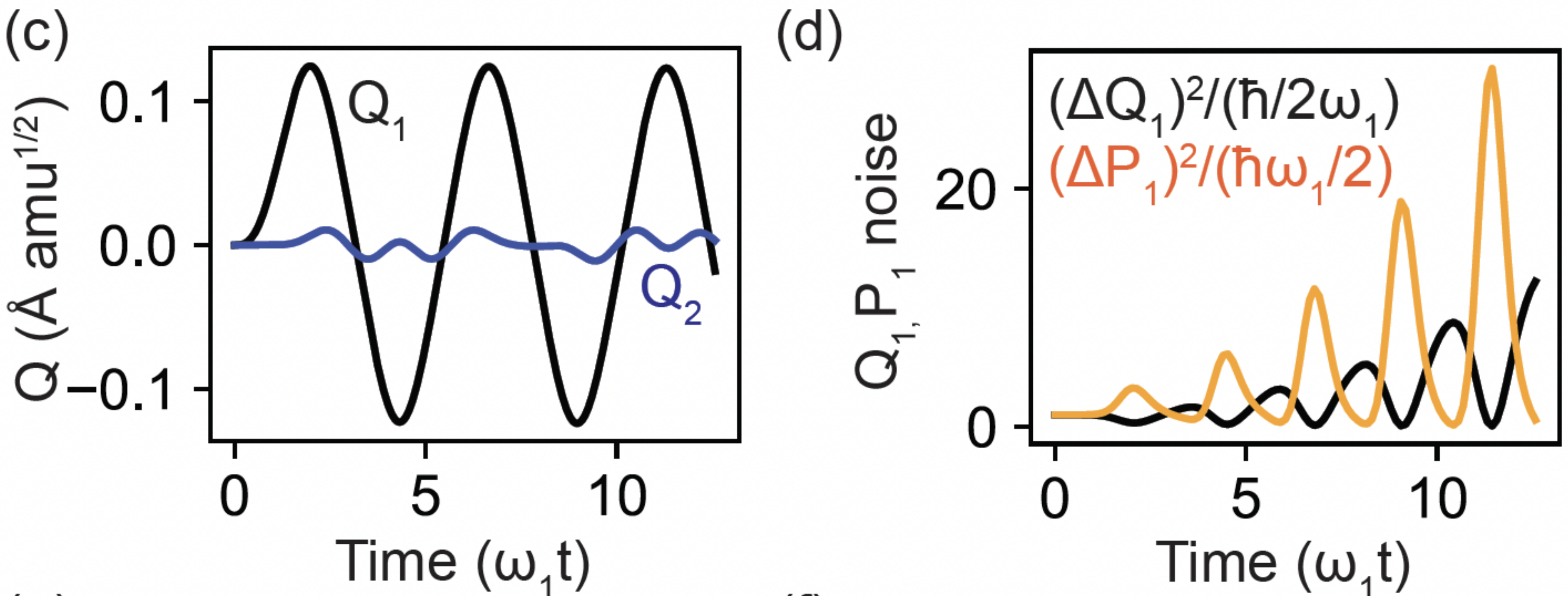

Generating quantum light states at optical frequencies

Goal: exploiting few-photon nonlinear optics to *deterministically* generate non-Gaussian quantum light states (e.g., Fock, Schrodinger cat, GKP states, etc.). Addressing the gap that such schemes don't exist for multiphoton states in *optics*.

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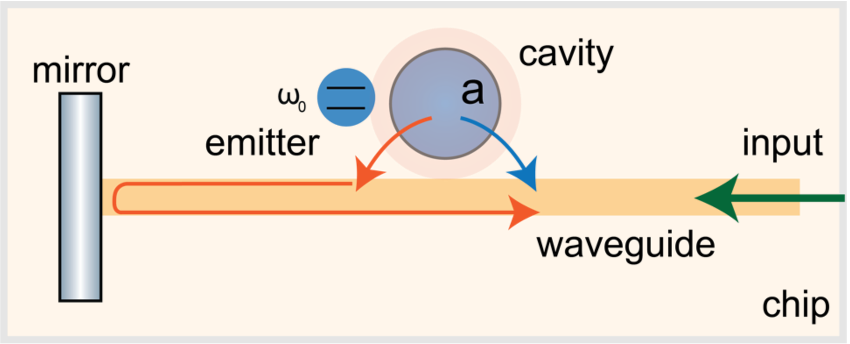
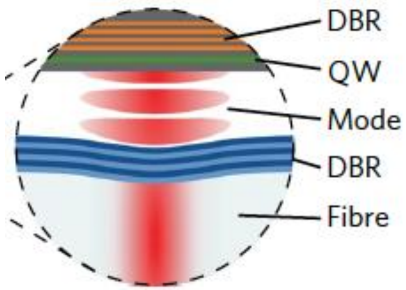
Dissipation engineering of few-photon nonlinear systems





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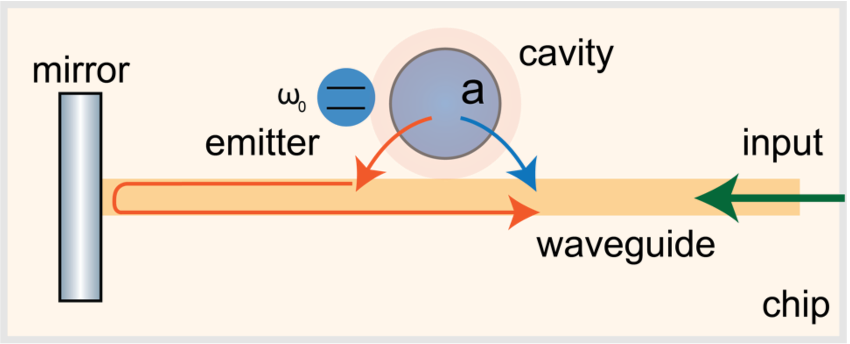
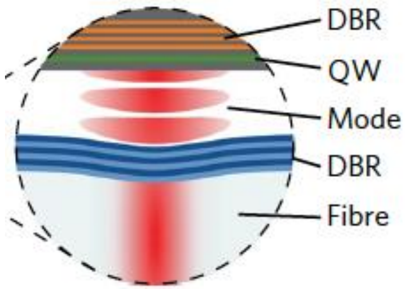
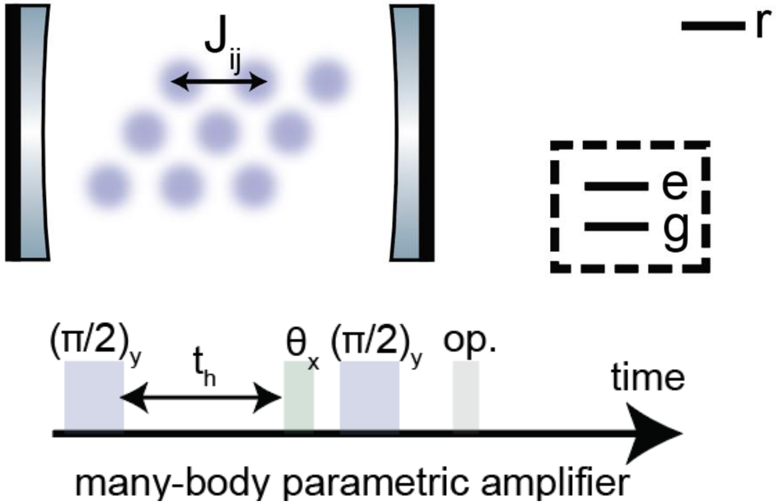




Engineering and transduction of multi-atom states



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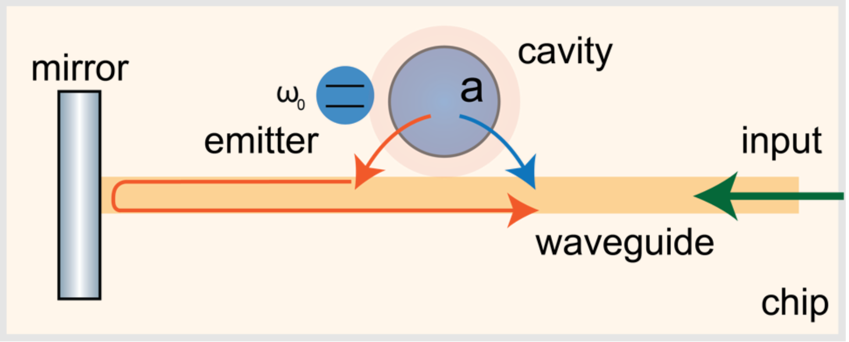

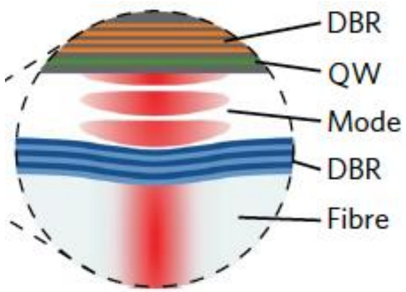

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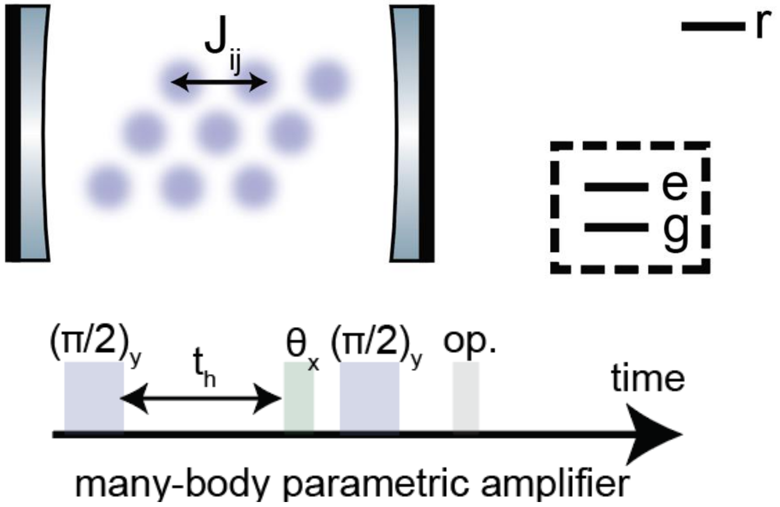

Then what? Generating certain non-Gaussian light states deterministically enables new photon-number resolving detectors, new sensing schemes, and the ability to fruitfully study driving and biasing systems with quantum light.

Controlling quantum noise in multimode nonlinear systems

Goal: understanding quantum noise in systems where many degrees of freedom interact (in the macroscopic regime). And, using this understanding to develop foundations of a next generation of low-noise light sources in different domains.

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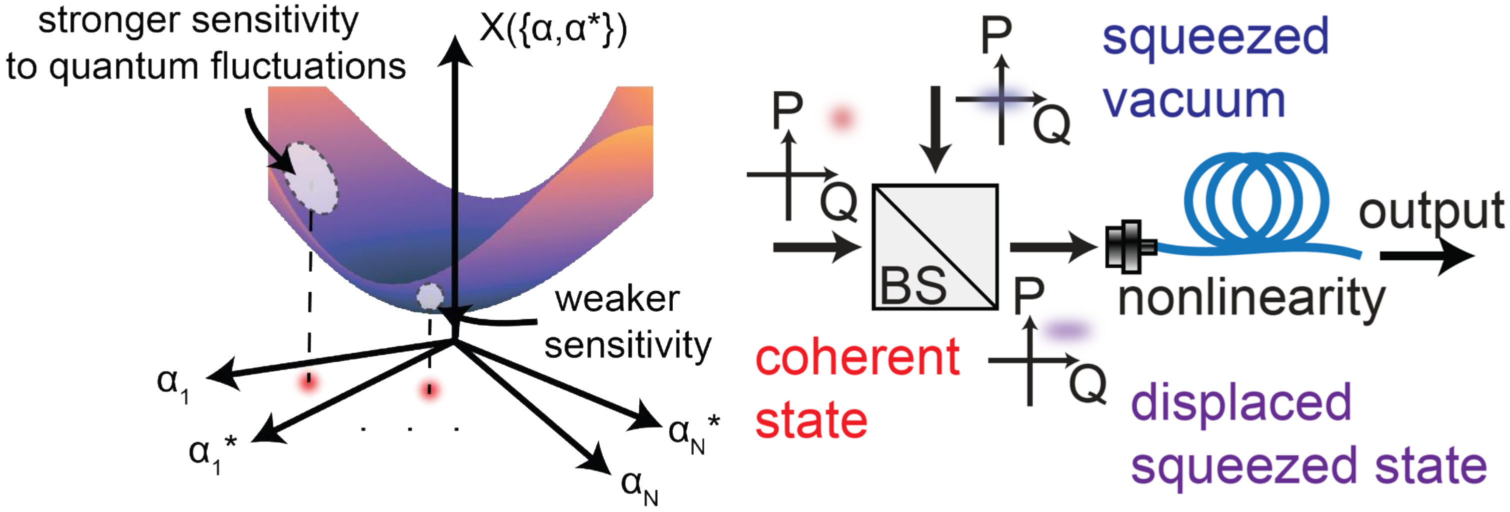
Optimal coherent and squeezed seeding



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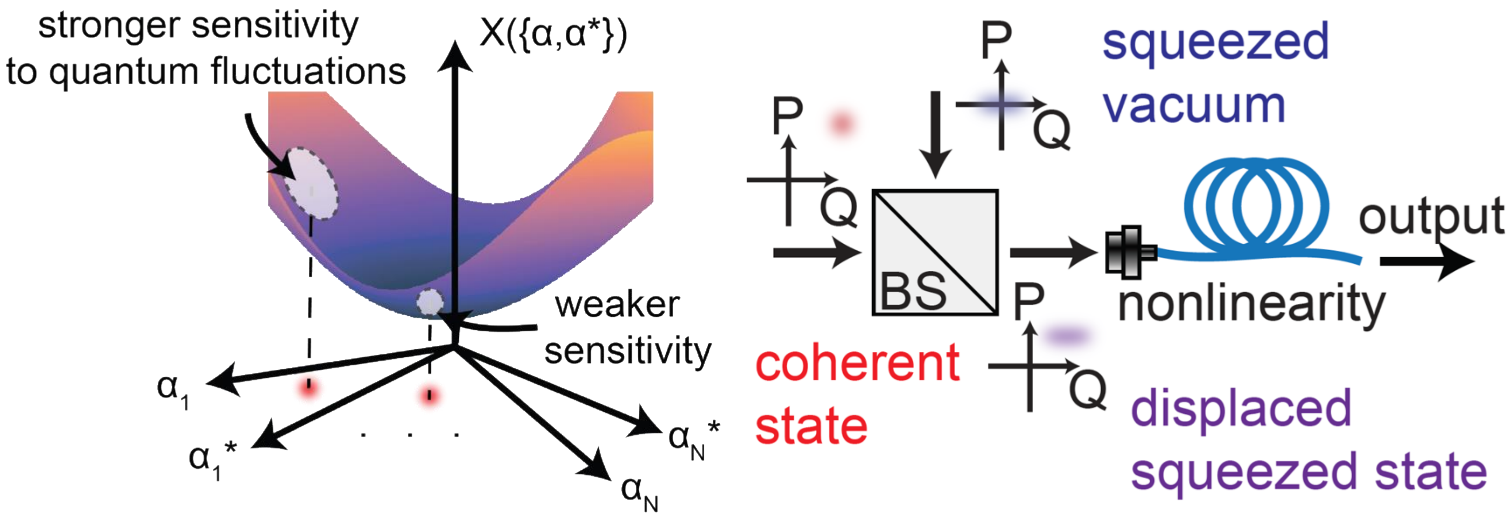

Then what? Optimal seeding schemes for noise-limited sources like X-ray lasers, theories of quantum noise in effects like high-harmonic generation, and realizing new detectors and imaging systems based on these sources

Acknowledgements

Collaborators (past and ongoing)

Prof. Marin Soljačić (MIT)
Prof. John Joannopoulos (MIT)
Prof. Ido Kaminer (Technion)
Prof. Prineha Narang (Harvard, UCLA)
Prof. Dmitri Basov (Columbia)
Prof. Liang Jie Wong (NTU Singapore)
Prof. Jelena Vuckovic (Stanford)
Prof. Marko Loncar (Harvard)
Dr. Charles Roques-Carmes (MIT, Stanford)
Dr. Shiekh Zia Uddin (MIT)
Dr. Yannick Salamin (MIT)
Jamison Sloan (MIT PhD)
Ali Ghorashi (MIT PhD)
Sean Chen (MIT undergrad)
Linh Nguyen (MIT undergrad)
and many, many others!

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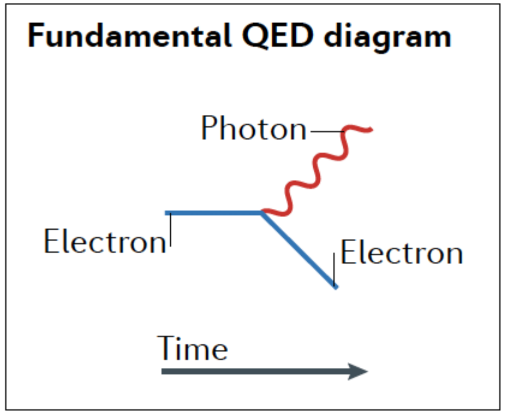
DOE CSGF
MIT
HARVARD UNIVERSITY
Air Force Office of Scientific Research
DARPA
Institute for Soldier Nanotechnologies

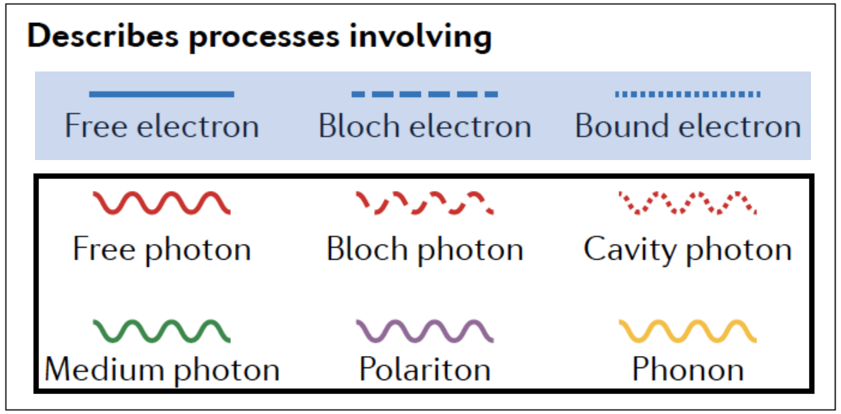
Discussion

Supplementary slides

Light-matter interactions with photonic quasiparticles

Macroscopic quantum electrodynamics (MQED)





Free electrons or bound electrons in atoms, molecules, solids, etc.

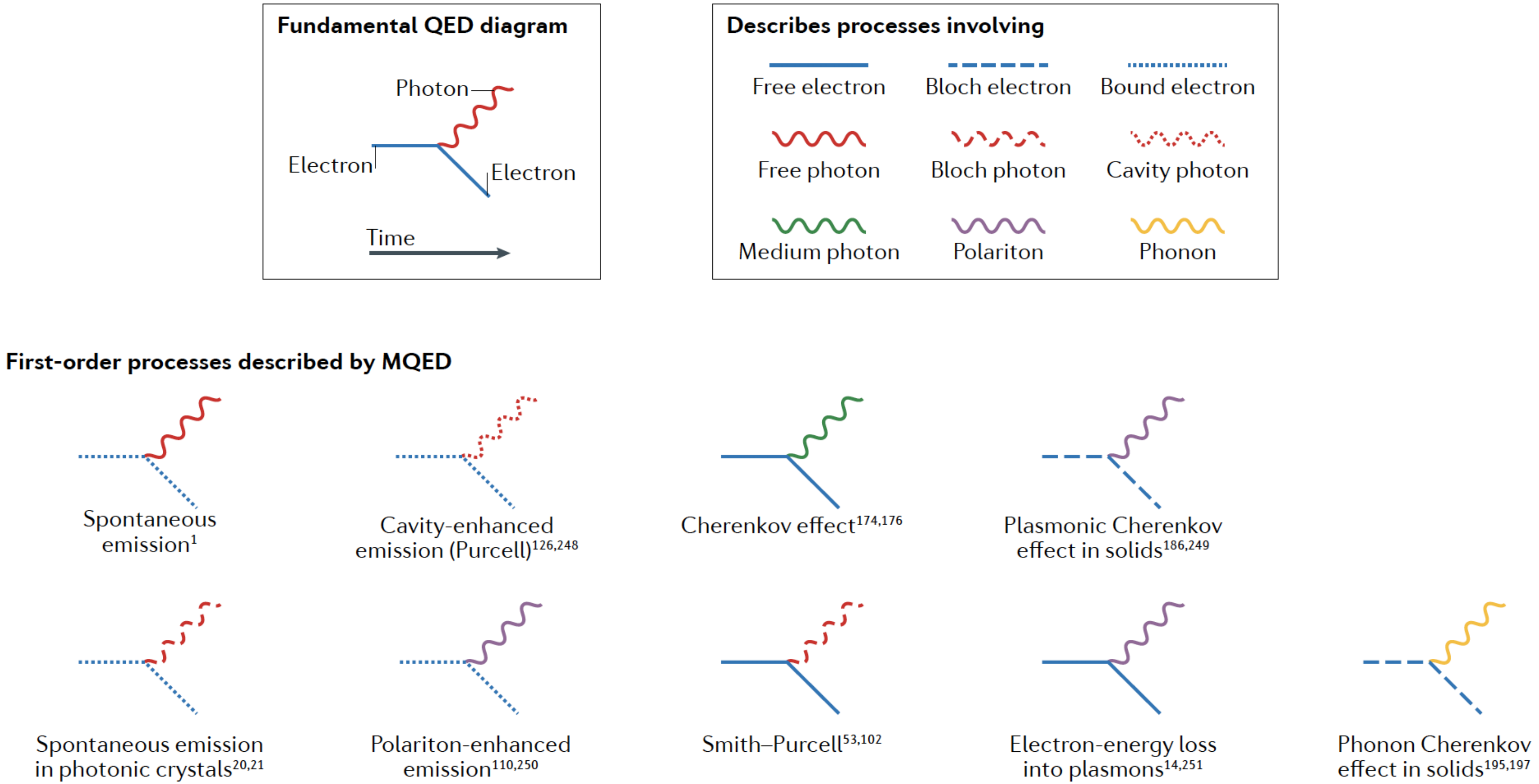
Coupling mediated by quantized EM field (e.g., vector potential)

$$\mathbf{A}(\mathbf{r},t) = \sqrt{\frac{\hbar}{\pi\epsilon_0}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \frac{\omega}{c^2} \int \mathrm{d}^3 r' \sqrt{\mathrm{Im}\ \epsilon(\mathbf{r}',\omega)}\ \left(\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \mathbf{f}(\mathbf{r},\omega) + \mathrm{h.c}\right)$$

loss — field propagator: dispersion/modes — polarization field operators

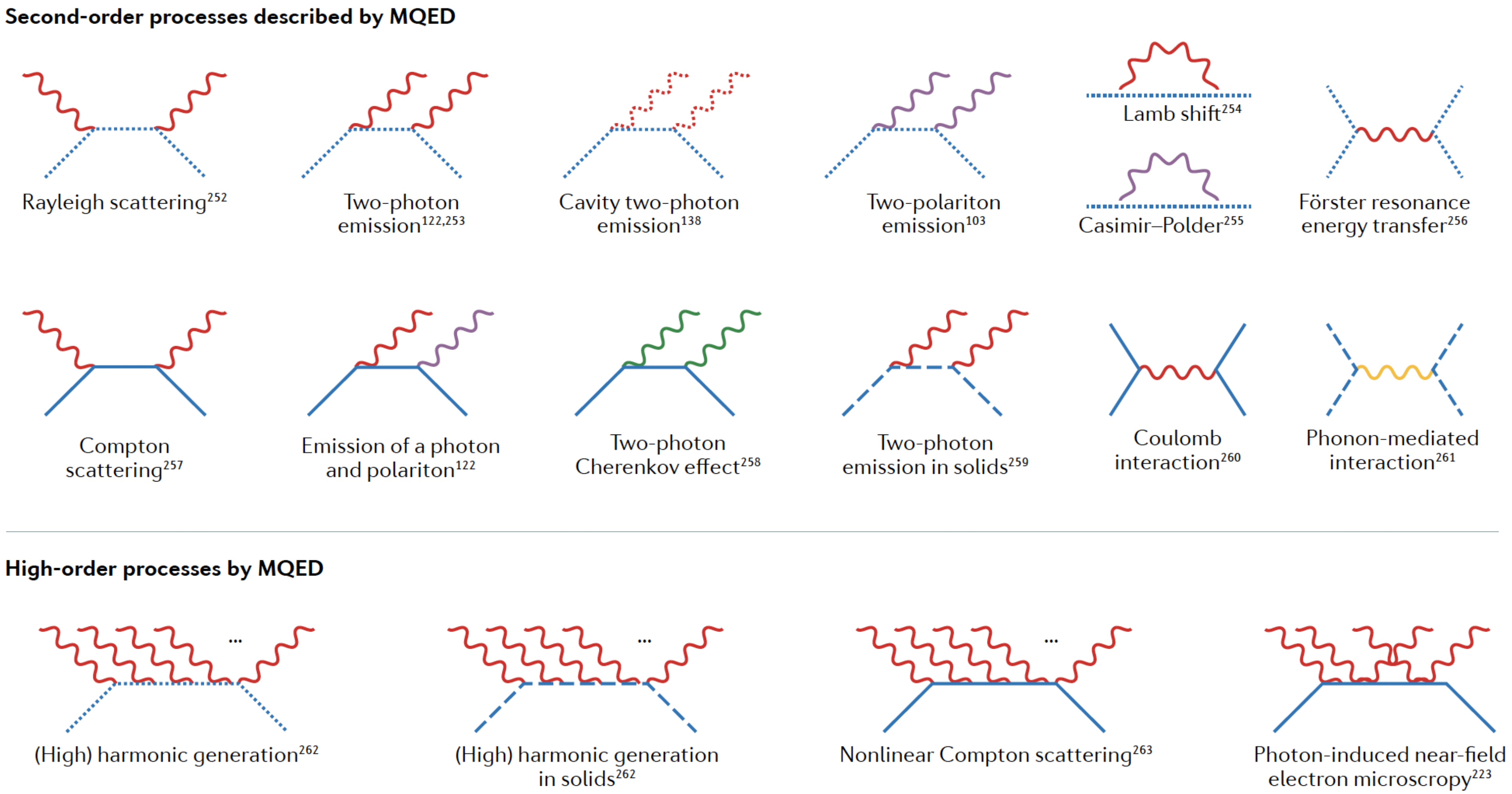
For applications, see: Buhmann *et al.* PRL (2008) *[Casimir-Polder forces],* Gonzales-Tudela *et al.* [*strong coupling*], **Rivera** *et al. Science* (2016), PNAS (2017) [*higher-order effects*], Goncalves *et al.* Nat. Comm. (2020) [*nonlocal plasmonics*], Sloan *et al.* PRL (2021) [*time-dependent materials*], Ben-Hayun *et al.* Sci. Adv. [*free electron radiation*], Svendsen *et al.* Nat. Comm. (2021), J. Chem. Theo. Comp. (2024) *[interfacing with DFT].*

Elementary QED processes with photonic quasiparticles



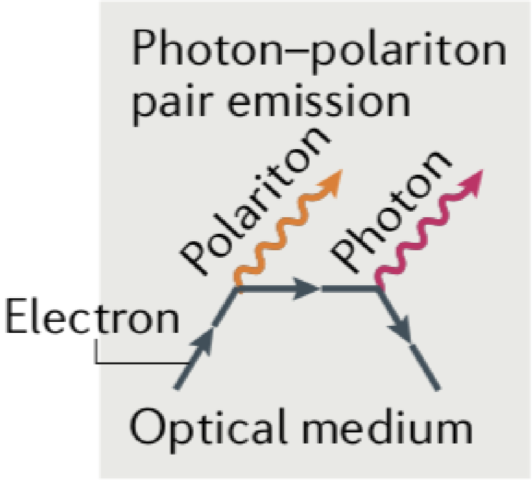
Rivera & Kaminer. *Nature Reviews Physics* (2020).

Elementary QED processes with photonic quasiparticles



Rivera & Kaminer. *Nature Reviews Physics* (2020).

Two-photon emission by a free electron into an X-ray and a polariton



$$\omega_{\mathrm{ph}} = \omega_{\mathrm{pol}} \frac{\beta n(\omega_{\mathrm{pol}}) \cos\theta_{\mathrm{pol}} - 1}{1 - \beta \cos\theta_{\mathrm{ph}}}$$

Rivera *et al.*, *Nature Physics* (2019).

Enhancing emission from fluctuating currents

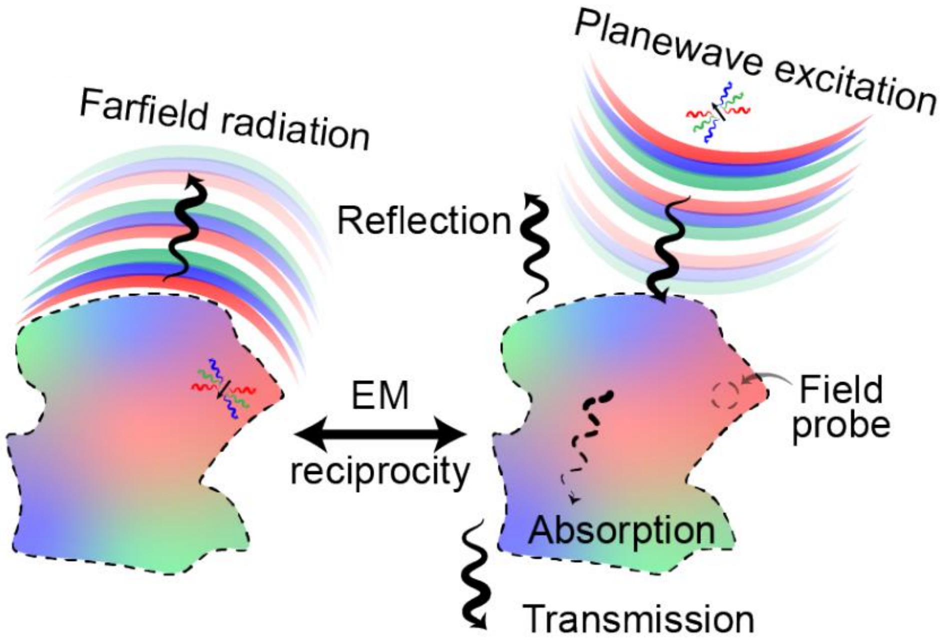
Special case: uniform spatial distribution of emitters in some volume:

$$\frac{dP}{d\omega d\Omega} = \frac{\pi}{\epsilon_0 \omega} \times \underbrace{S(\omega)}_{\text{spectral function}} \times \underbrace{(V_{eff}(\omega)/\lambda^3)}_{\text{proportional to absorption}}$$

spectral function

proportional to absorption

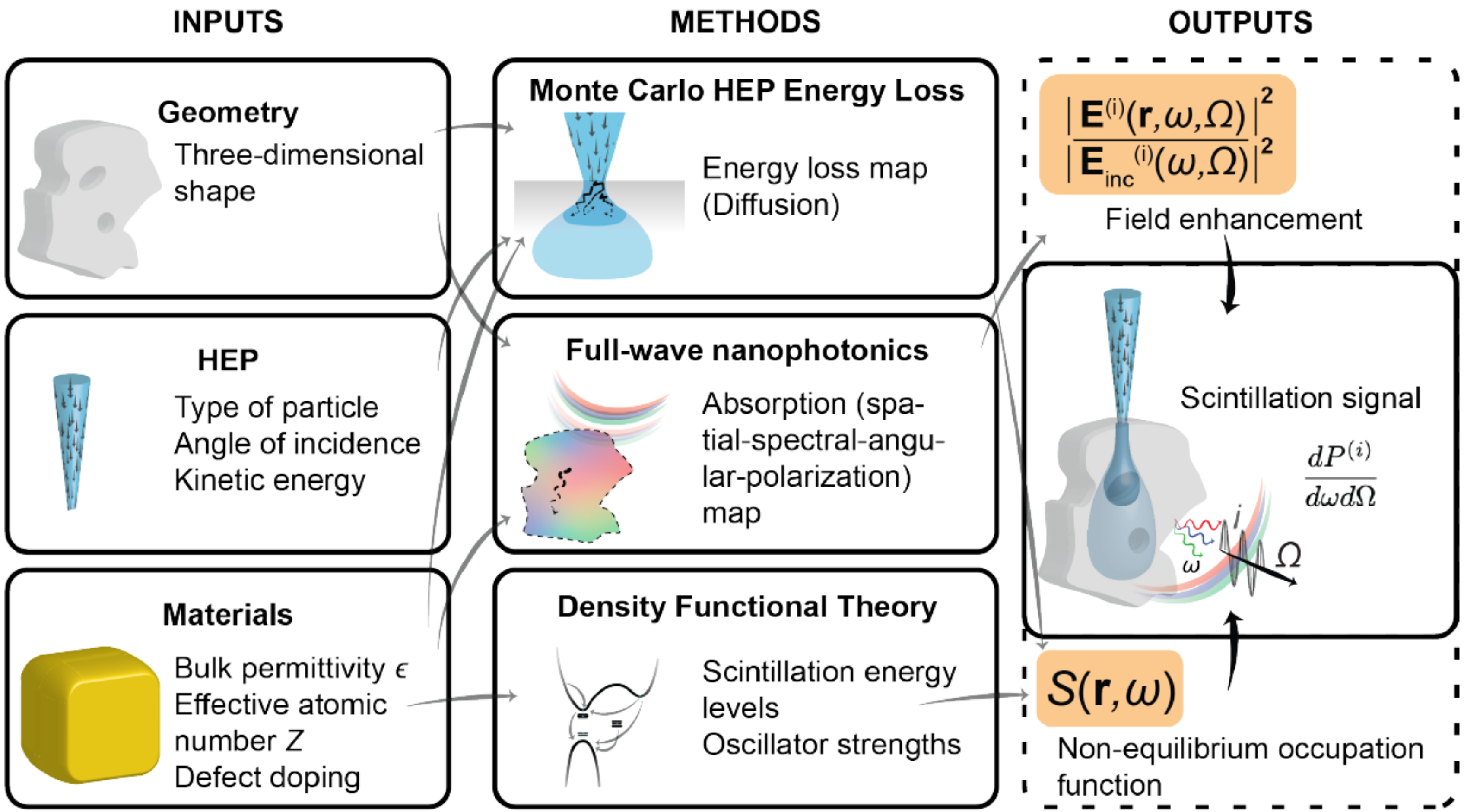
$$\sum_i \int d^3r' \left| \frac{\mathbf{E}(\mathbf{r}', \mathbf{r}, i, \omega)}{E_{inc}(\mathbf{r}', \mathbf{r}, i, \omega)} \right|^2$$



Connection to “absorbed power” means faster simulations, inverse design and optimization, and ability to borrow designs from enhanced solar cells/LEDs/thermal emission.

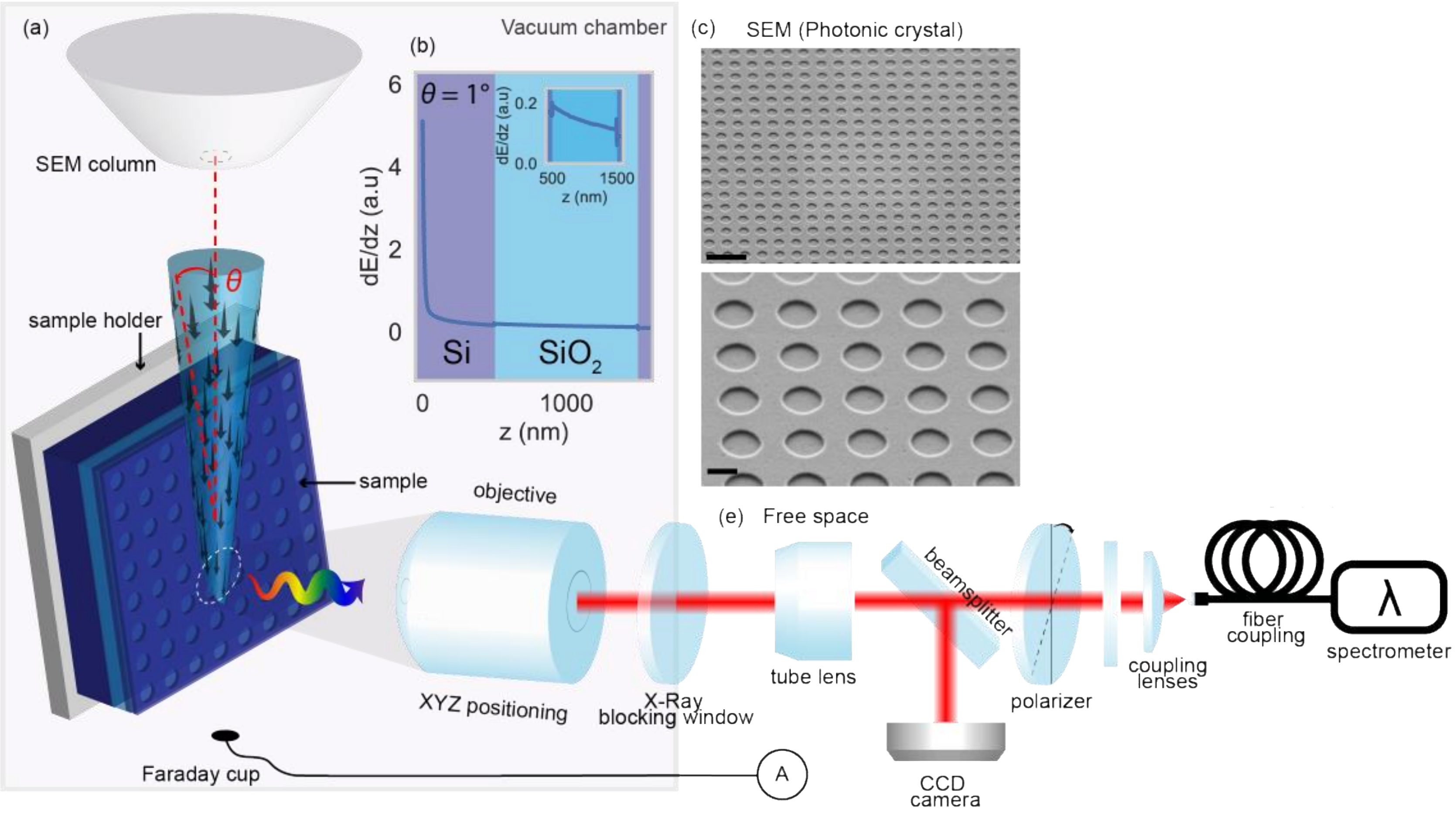
Roques-Carmes* and **Rivera*** *et al.* Science (2022)

The end-to-end numerical pipeline for nanophotonic scintillation, summarized



Roques-Carmes* & **Rivera*** *et al.* Science (2022).

Experiment I: enhancing scintillation induced by electrons



Roques-Carmes* & **Rivera*** *et al.* Science (2022).

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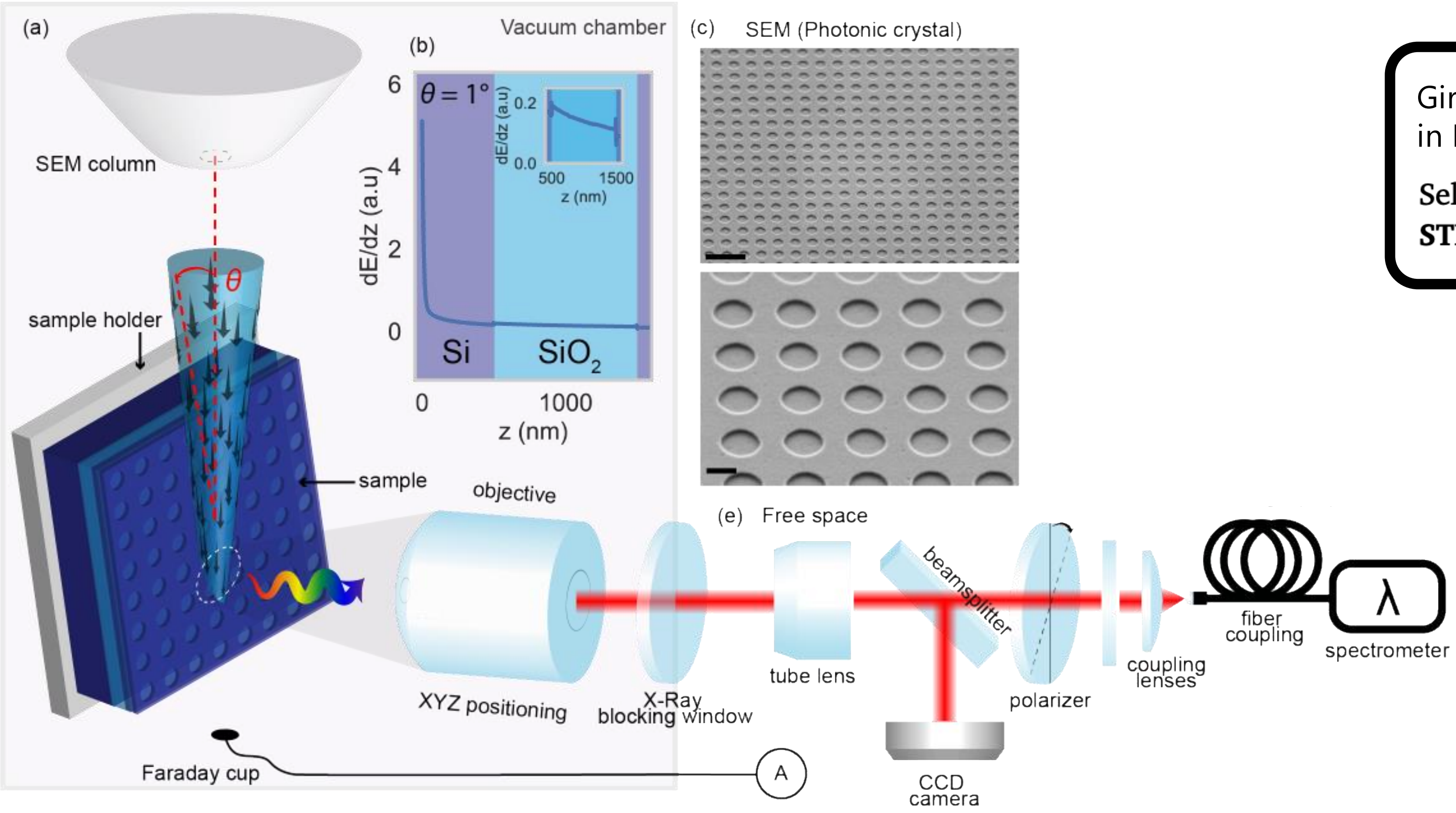
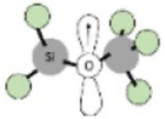
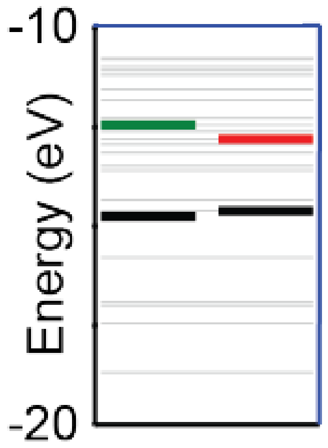


Roques-Carmes* & **Rivera*** *et al.* Science (2022).

Girard et al., Reviews in Physics (2019)		**Optical transition (bandwidth) [eV]**
Self-trapped hole STH_1		2.61 (1.2) 1,88 (0.2–0.5)

[DFT energy levels]



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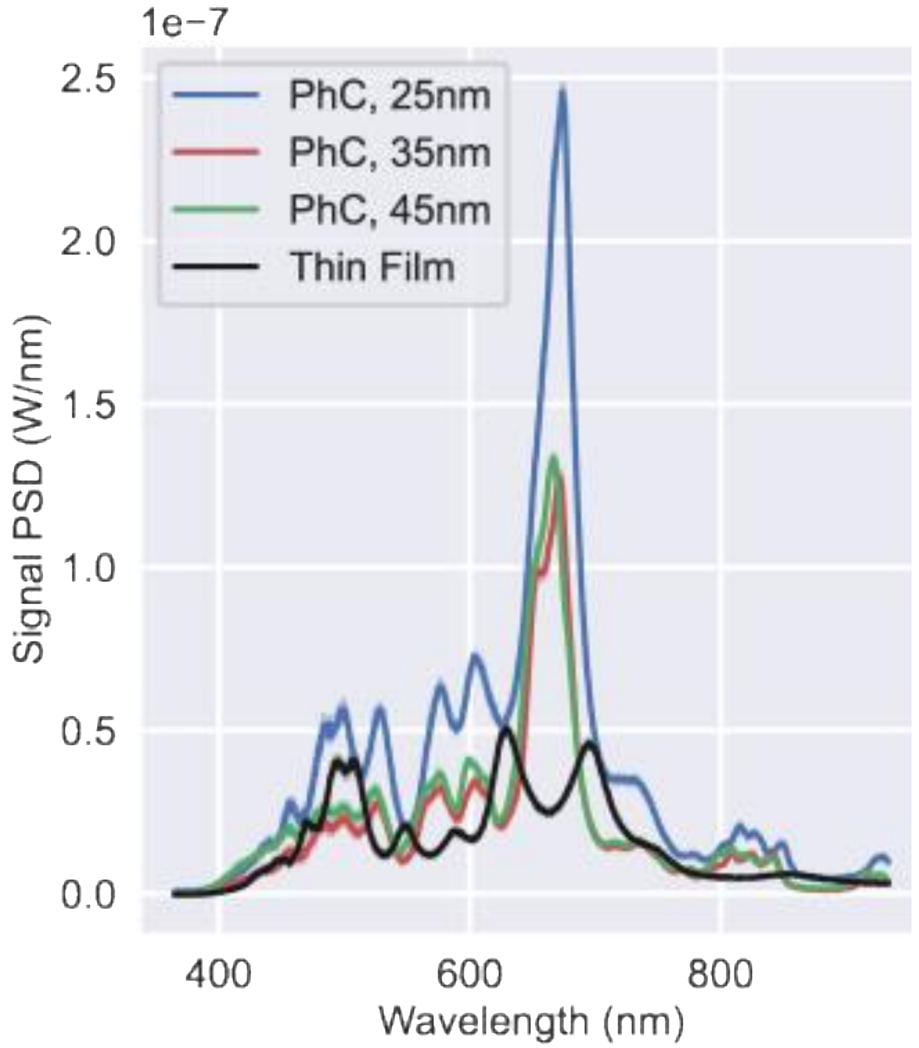
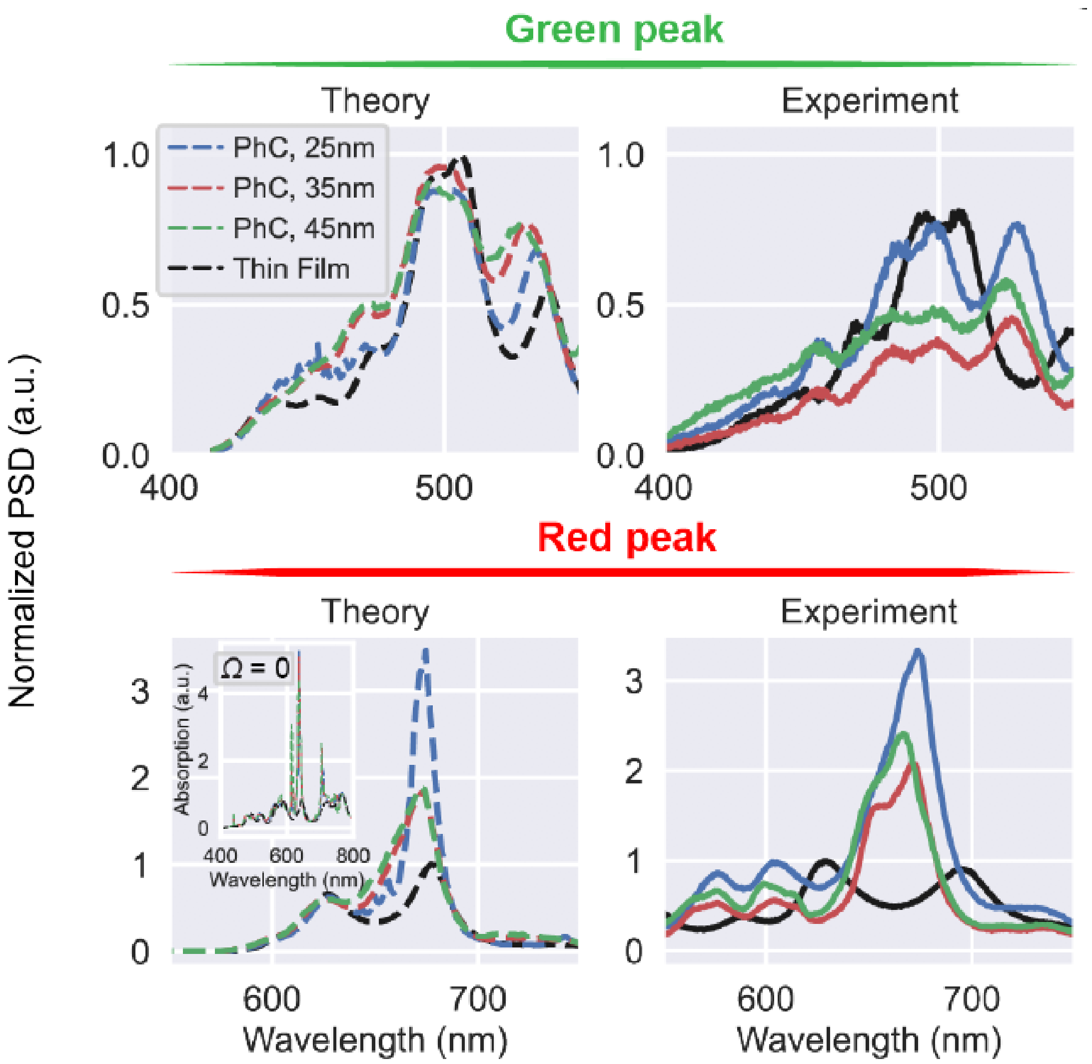
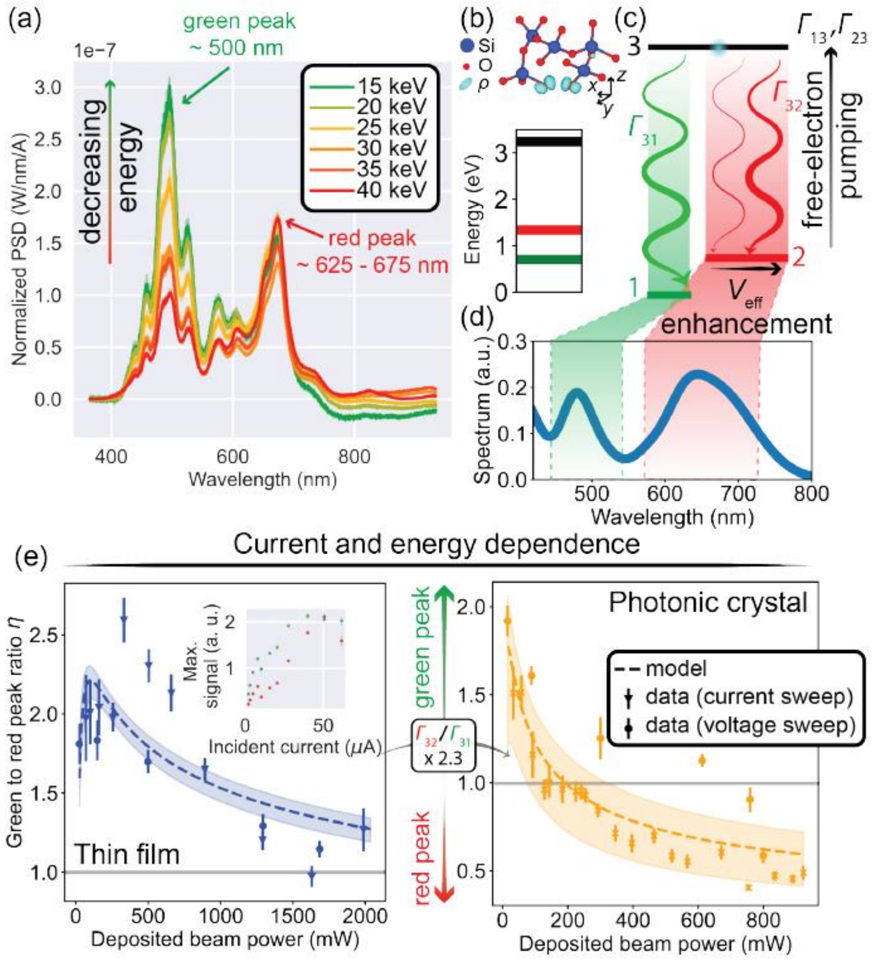




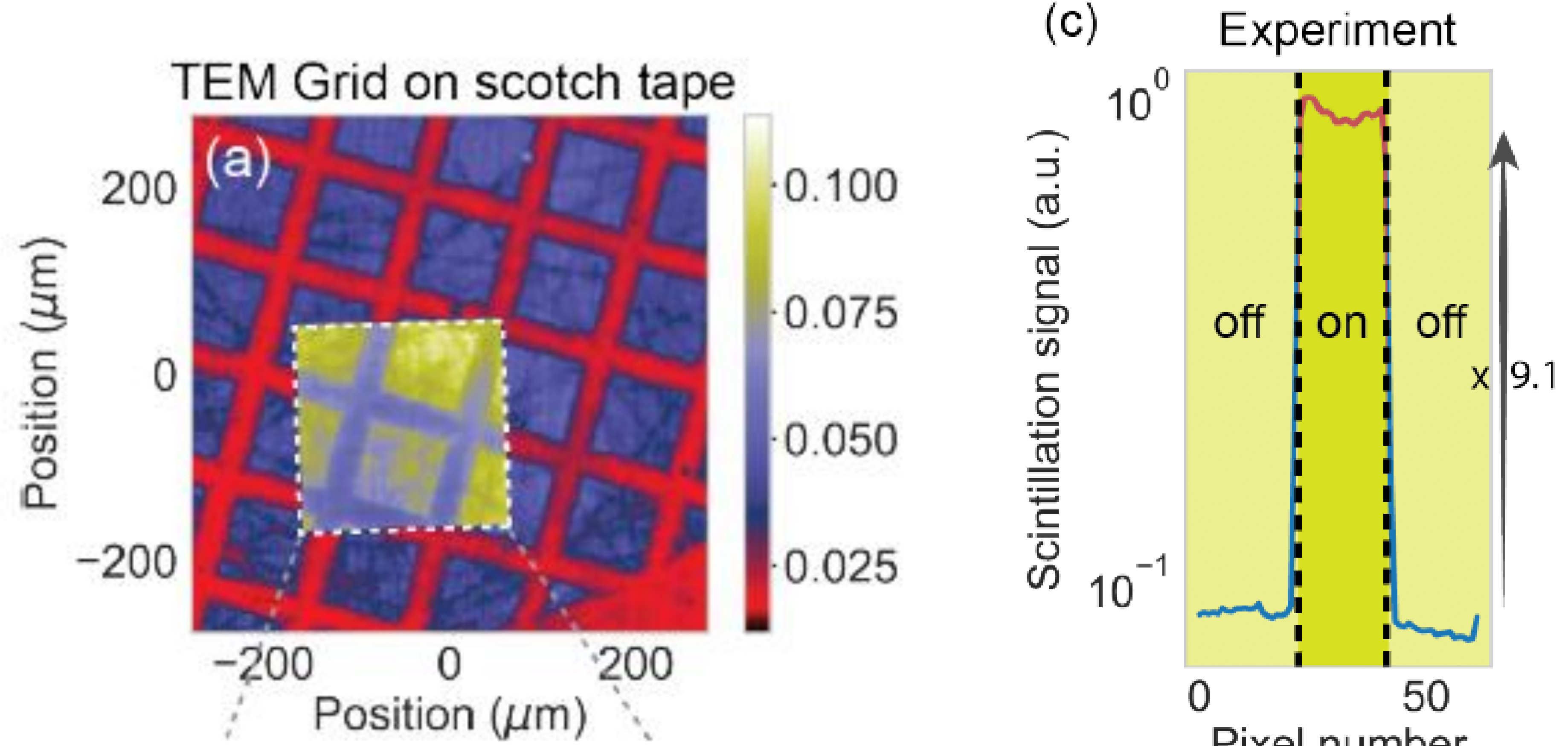
Out-of-equilibrium kinetics of defects controlled by nanophotonics



- Beyond predicting "nanophotonic" features the framework also, accounts for features requiring microscopics+nanophotonics
  - Nonlinear dependence of emission on pump + "cross-over" from green to red
    - 3-level dynamics of self-trapped holes

$$\begin{cases} \frac{dp_1}{dt} = -\Gamma_{13}\, p_1(1-p_3) + \Gamma_{31}\, p_3(1-p_1) \\ \frac{dp_2}{dt} = -\Gamma_{23}\, p_2(1-p_3) + \Gamma_{32}\, p_3(1-p_2) \\ \frac{dp_3}{dt} = \Gamma_{13}\, p_1(1-p_3) - \Gamma_{31}\, p_3(1-p_1) \\ \qquad +\Gamma_{23}\, p_2(1-p_3) - \Gamma_{32}\, p_3(1-p_2) \end{cases}$$

Experiment: enhancing scintillation induced by X-rays



Also did this for electron-induced scintillation: 3-6-fold enhancement.

Roques-Carmes* and **Rivera*** *et al.* Science (2022)

Some immediate results of quantum sensitivity analysis

Coherent state inputs: $(\Delta X)^2 = ||\nabla_{\boldsymbol{\alpha}(0)} X(t)||^2$

Correlation functions: $\langle \delta X \delta Y \rangle = (\nabla_{\boldsymbol{\alpha}(0)} X(t))^* \cdot (\nabla_{\boldsymbol{\alpha}(0)} Y(t))$

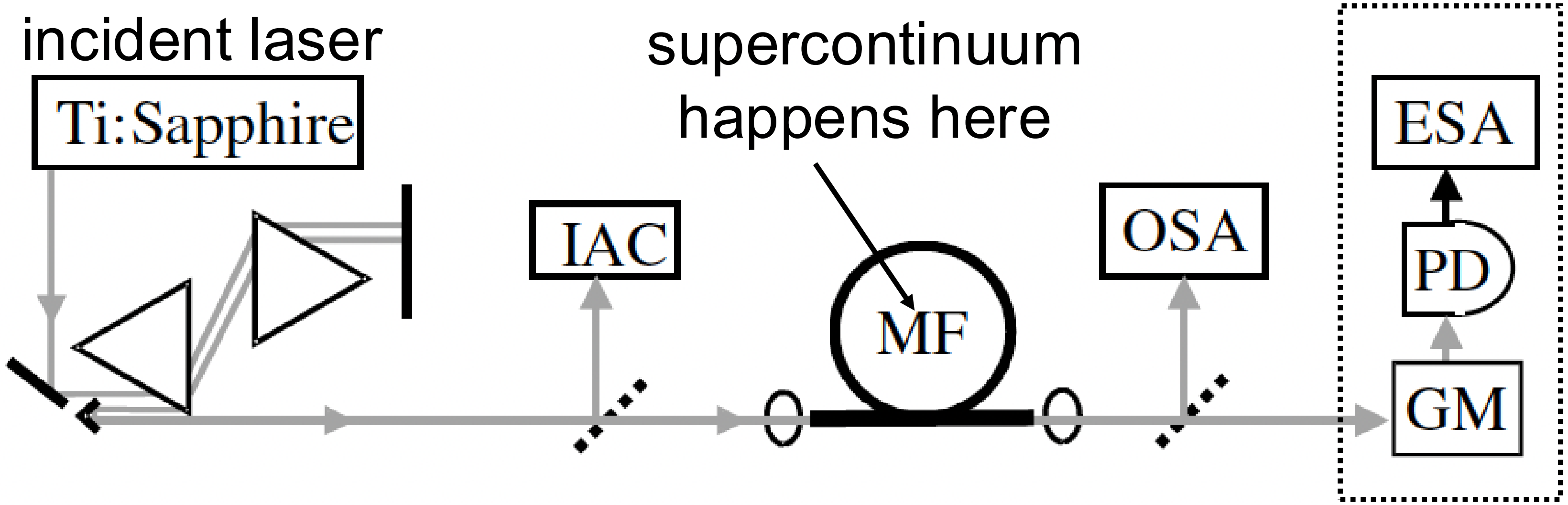
Other degrees of freedom (gain, phonons, etc.):

$$(\Delta X)^2 = ||\nabla_{\boldsymbol{\alpha}(0)} X(t)||^2 + ||\nabla_{\boldsymbol{\beta}(0)} X(t)||^2$$

Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Quantum fluctuations in supercontinuum generation

Supercontinuum is famously a noisy process, and has a fundamental quantum origin: amplification of incident vacuum noise.



Corwin *et al.* *PRL* (2003).

Nearly 1-10% pulse-to-pulse fluctuation from inputs with <0.01%. Individual wavelengths are not in coherent states, even when initial light is!

The classical description of supercontinuum generation

Pulse evolution: the generalized nonlinear Schrodinger equation

$$\partial_z A(z,t) = \underbrace{-\frac{\alpha}{2}A(z,t)}_{\text{loss}} + \underbrace{\sum_{k=2}^{\infty}\frac{i^{k+1}\beta_k}{k!}\partial_t^k A(z,t)}_{\text{dispersion: } k = k(\omega)} + \underbrace{i\gamma A(z,t)\int dt' R(t')|A(z,t-t')|^2}_{\text{Nonlinearity: instantaneous (electronic) + delayed (phonons)}}$$

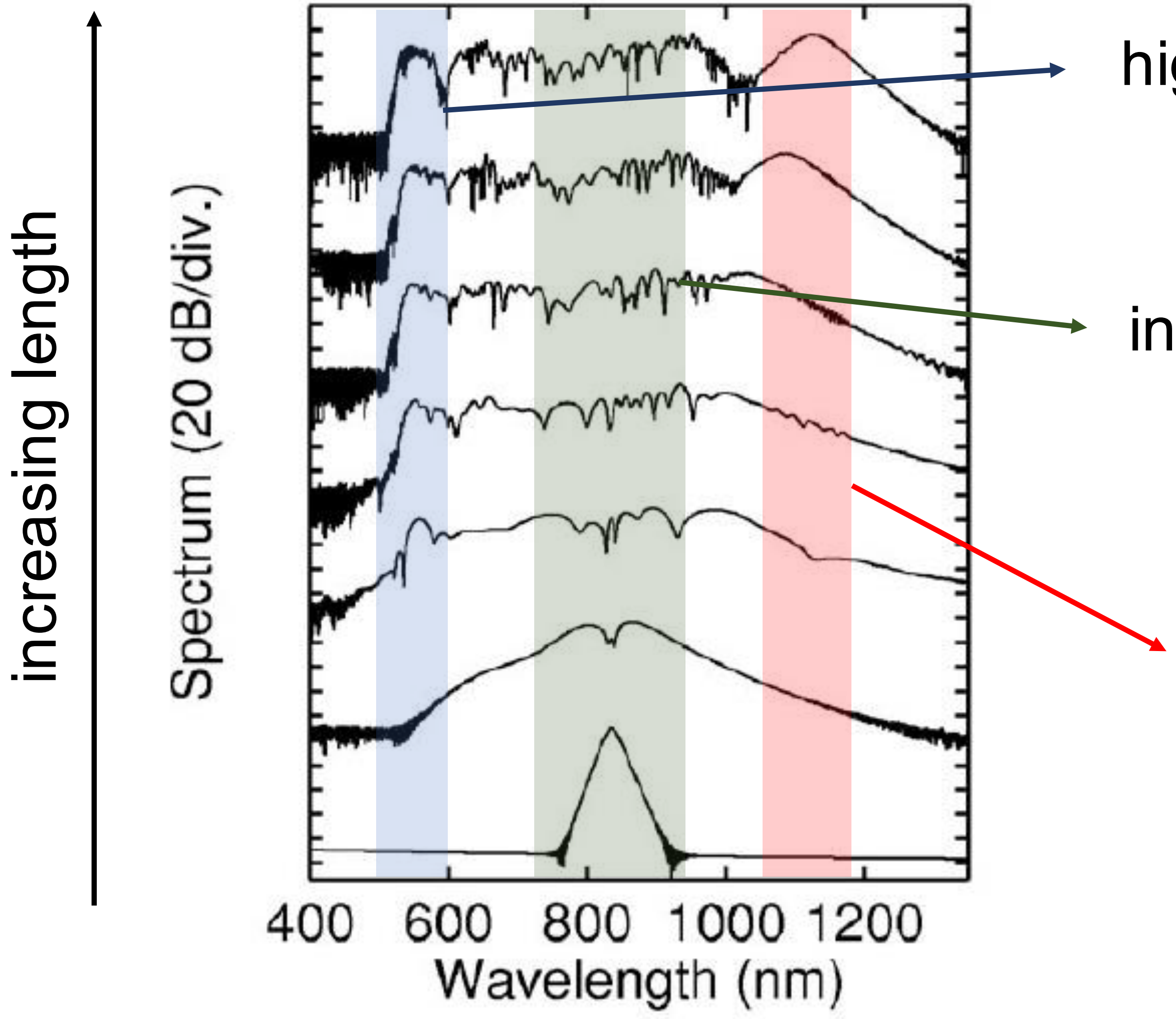
loss

dispersion:
k = k(ω)

Nonlinearity: instantaneous (electronic)
+ delayed (phonons)

This equation (+ variants) has been studied by many authors for a long time (50+ years)!

The physical processes leading to supercontinuum generation



high-order dispersion + nonlinearity

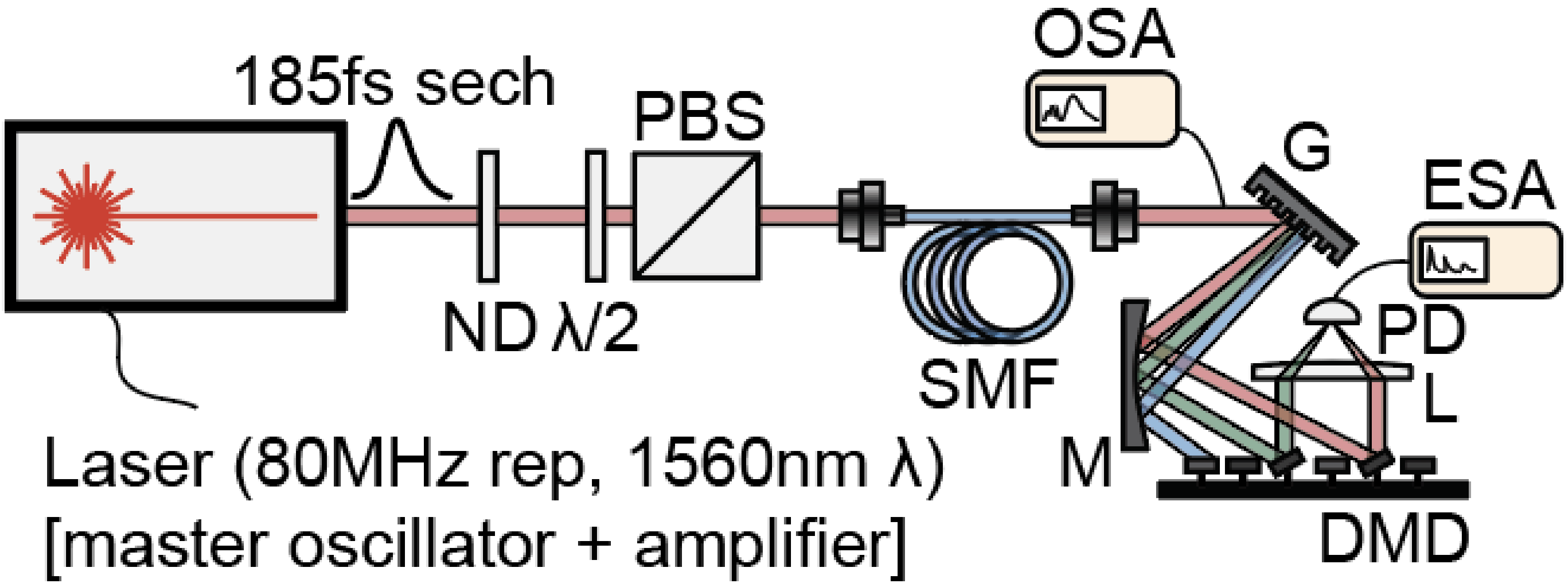
instantaneous part of nonlinearity

soliton dynamics (dispersion + instantaneous nonlinearity) and Raman scattering

Dudley *et al. Rev. Mod. Phys.* (2006).

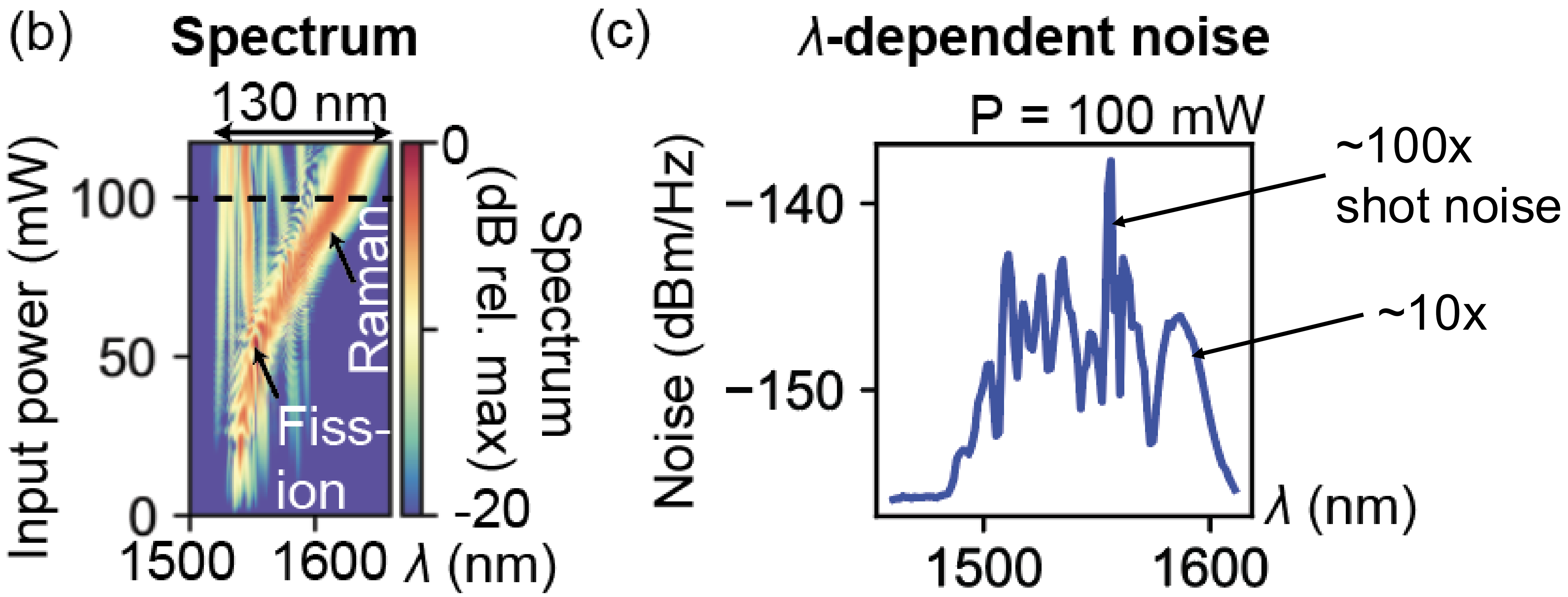
A new low noise regime in supercontinuum generation

(a) **Experimental schematic**



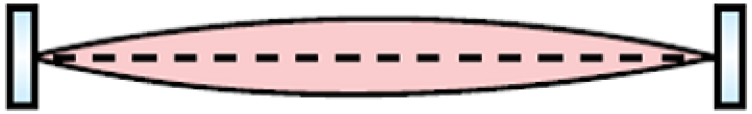
Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

A new low noise regime in supercontinuum generation

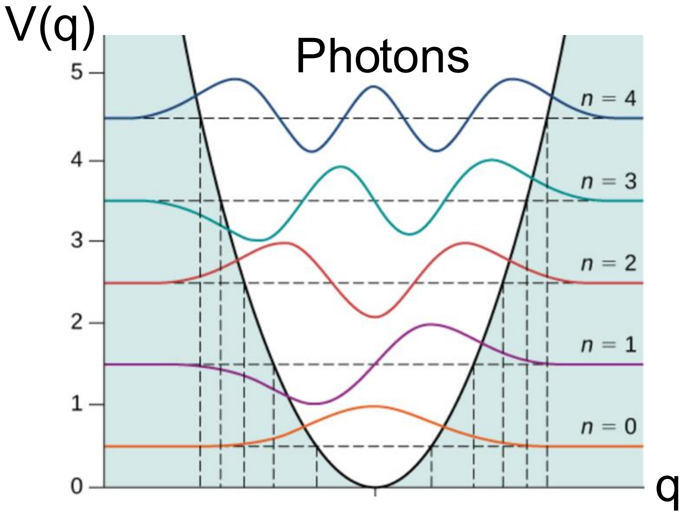


Uddin* and **Rivera*** *et al.* arXiv:2311.05535 (2023)

Fock states are fundamental, broadly applicable, and difficult to create



$$A(z,t) = 2\sqrt{\frac{\hbar\omega}{\epsilon_0 V}}\sin(kz)q$$



Fock states are the most fundamental states of light, and yet, extremely difficult to produce

- In optics, $n \geq 2$ is difficult to produce
- Proposed techniques are generally non-deterministic

Applied

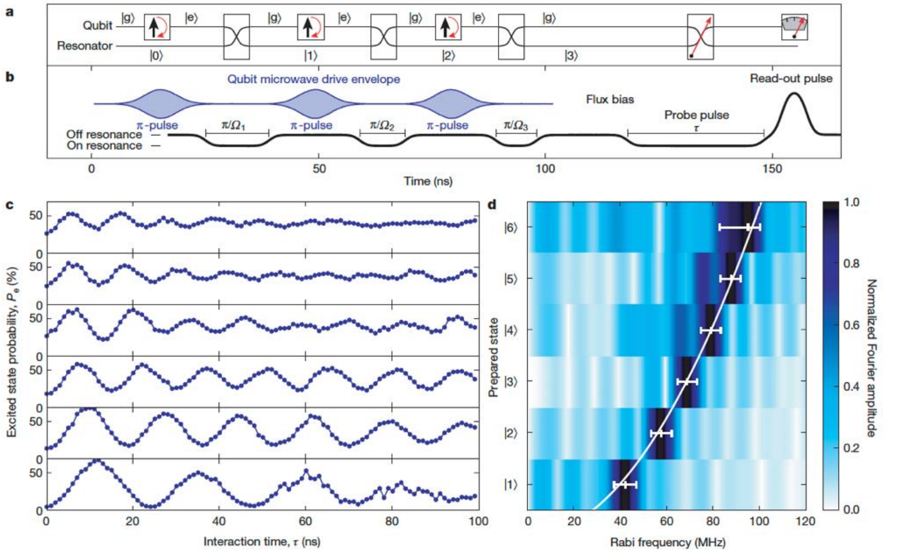
- Quantum algorithms
- Optical quantum computing
- Metrology & spectroscopy

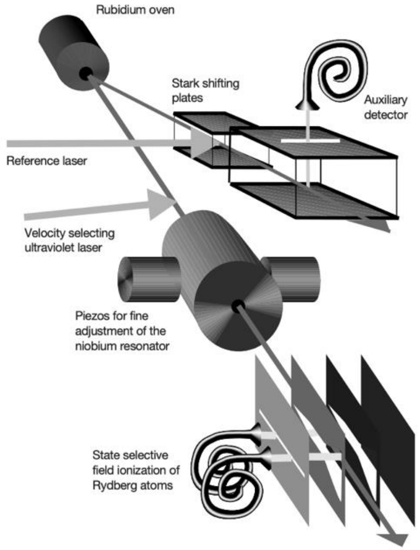
Fundamental

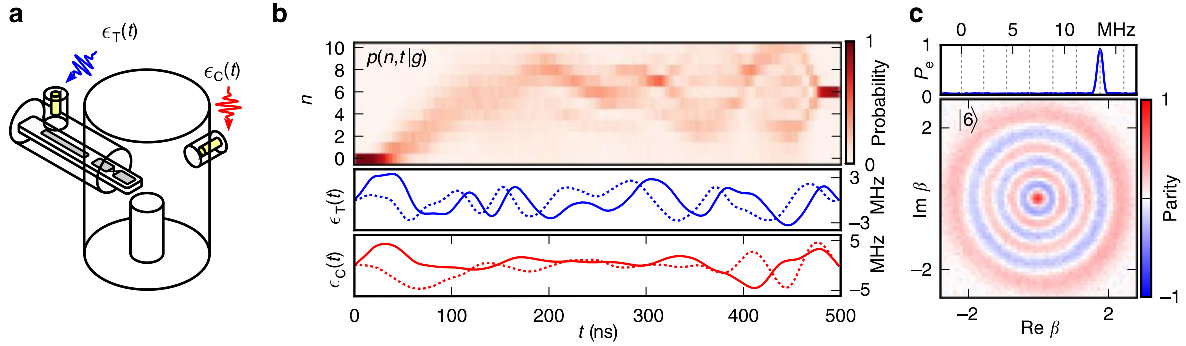
- Most basic state of light
- No classical counterpart
- Workhorse for QM states
- Macroscopic quantum light

How Fock states are currently generated

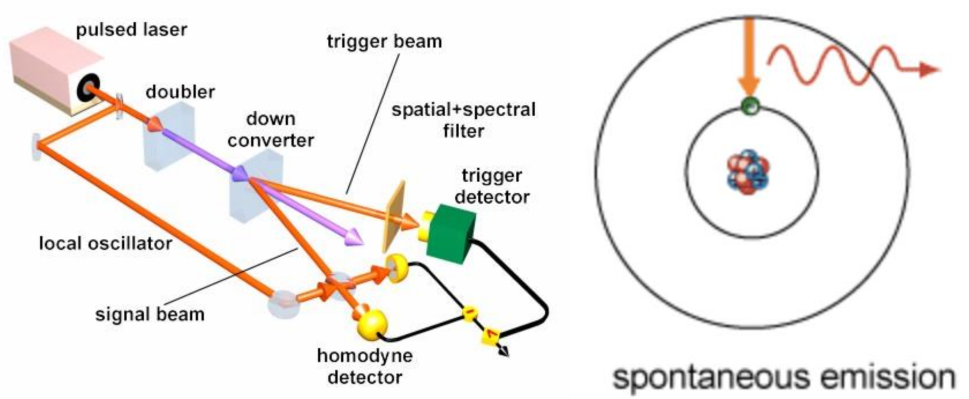
Microwave Fock states in resonators

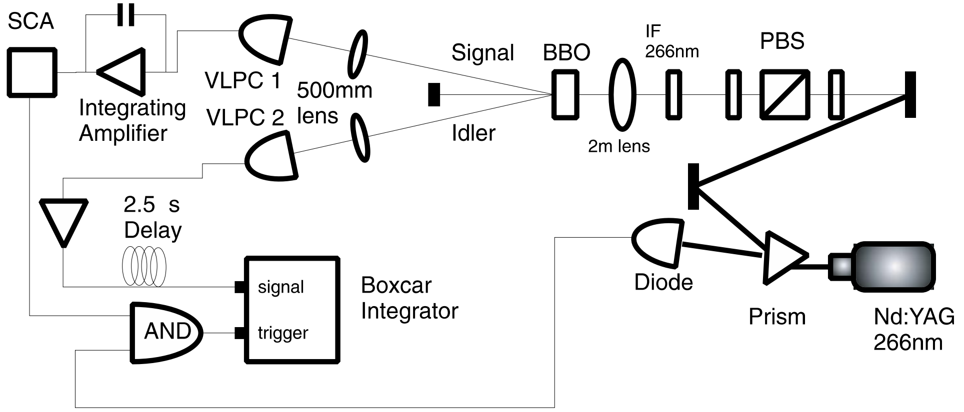






Propagating optical Fock states

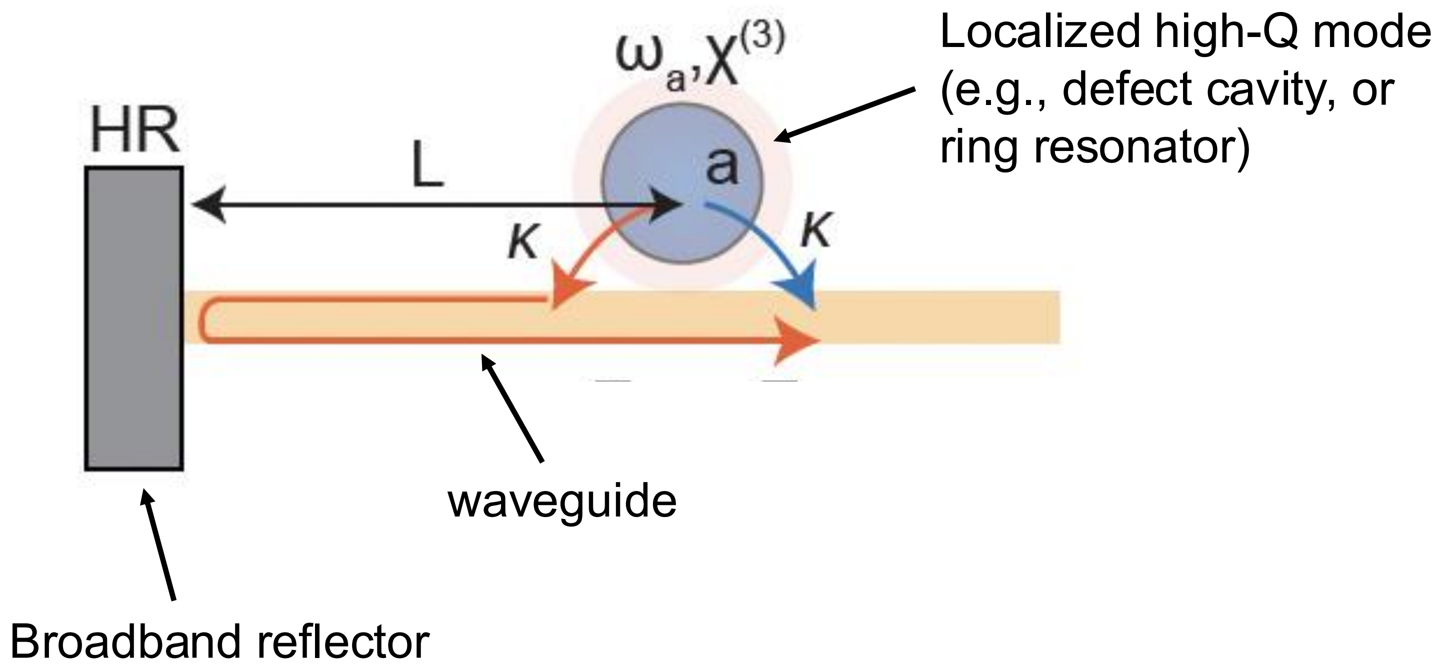




Varcoe *et al.* Nature (2000). Hofheinz *et al. Nature* (2008). Heeres *et al. Nat. Comm.* (2017). Bimbard *Nat. Phot.* (2010). Waks *et al. New J. Phys.* (2006).

Example: a nanophotonic structure with strong dependence of loss on index

A cavity with a nonlinear resonant mirror



Outcoupled field:

$$\sqrt{\kappa} + \pm\sqrt{\kappa} \times e^{i\omega_a(n)L/v} \times (-i)$$

Lossless condition:

$$ie^{i\omega_a(n)L/v} = \pm 1$$

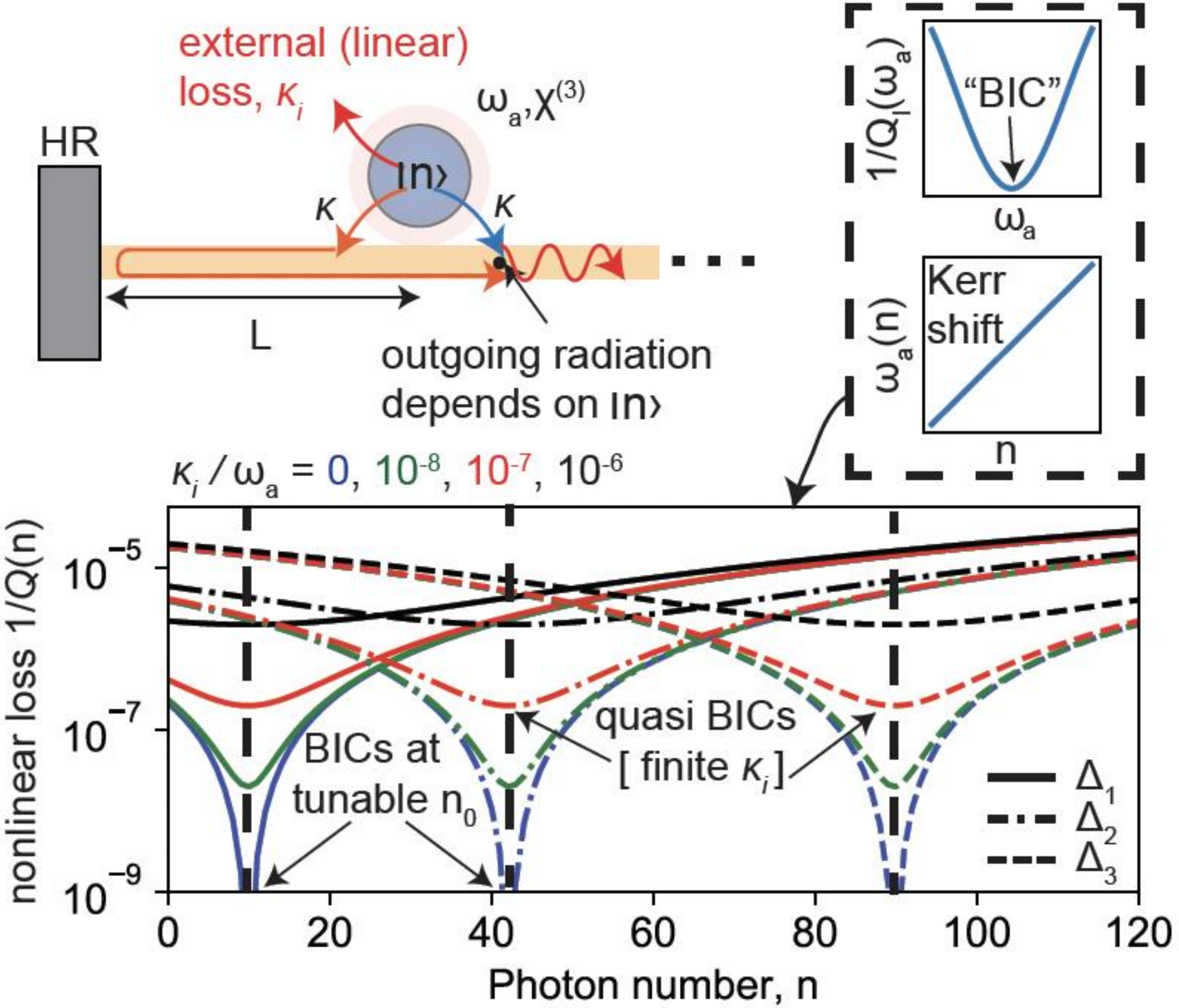
Nonlinear loss:

$$Q = Q(\omega_a(n))$$

Frequency-dependent loss
Hsu *et al.* Nature (2013)
Hsu *et al.* Nat. Rev. Mat. (2016)
Yu *et al.* Nat. Phot. (2017)
Yang *et al.* Nat. Phot. (2020)
Yu *et al.* Nat. Phot. (2021)

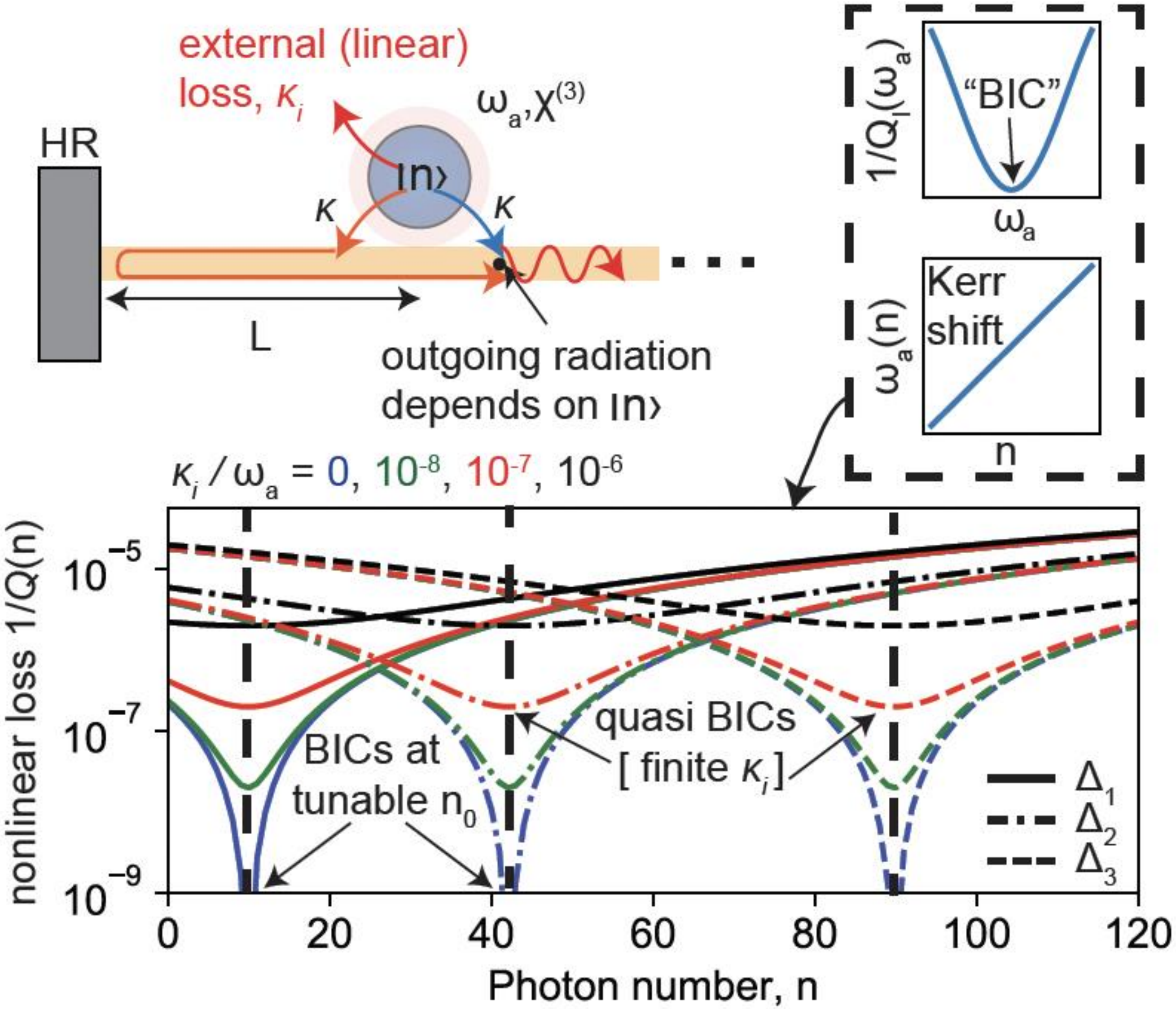
Rivera* & Sloan* *et al.* arXiv:2211.01514

Nonlinear loss and Fock stable states



Rivera* & Sloan* *et al.* arXiv:2211.01514

Nonlinear loss and Fock stable states



A general formalism to predict quantum state dynamics of driven-dissipative nanophotonic systems with nonlinear loss:

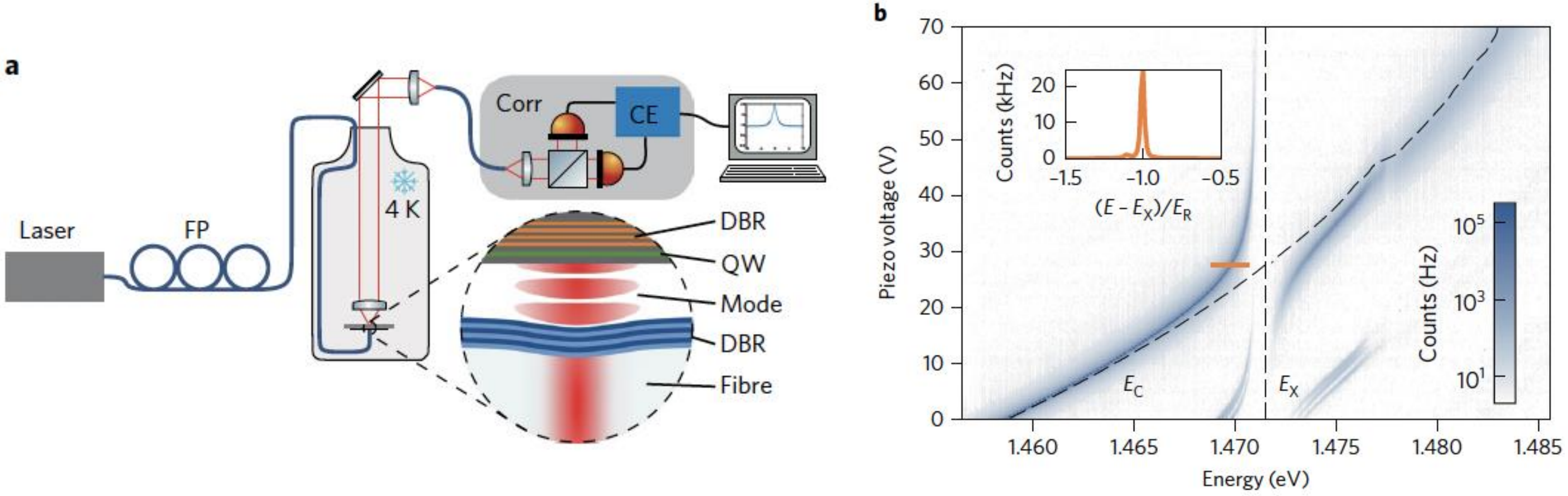
$$\dot{\rho} = -i[H_K + H_{\text{drive}}, \rho] + \mathcal{D}[\rho]$$

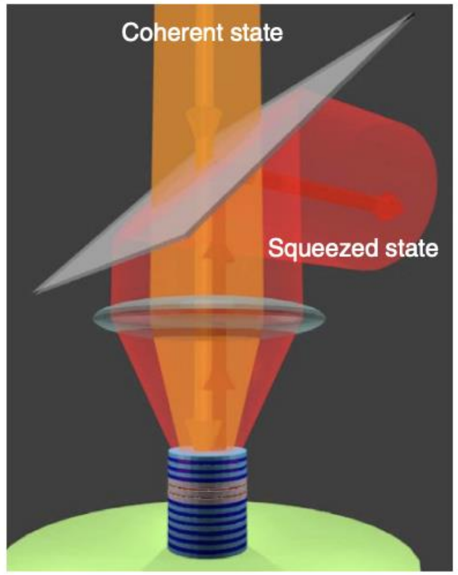
Nonlinear dissipator:

$$\langle m|\mathcal{D}[\rho]|n\rangle = -\left(mK_l(\omega_{m,m-1}) + nK_l^*(\omega_{n,n-1})\right)\rho_{m,n} + \sqrt{(m+1)(n+1)}\left(K_l(\omega_{m+1,m}) + K_l^*(\omega_{n+1,n})\right)\rho_{m+1,n+1}$$

Energy to add a photon to a nonlinear cavity

Extremely strong Kerr nonlinearities from semiconductors (excitons) in microcavities



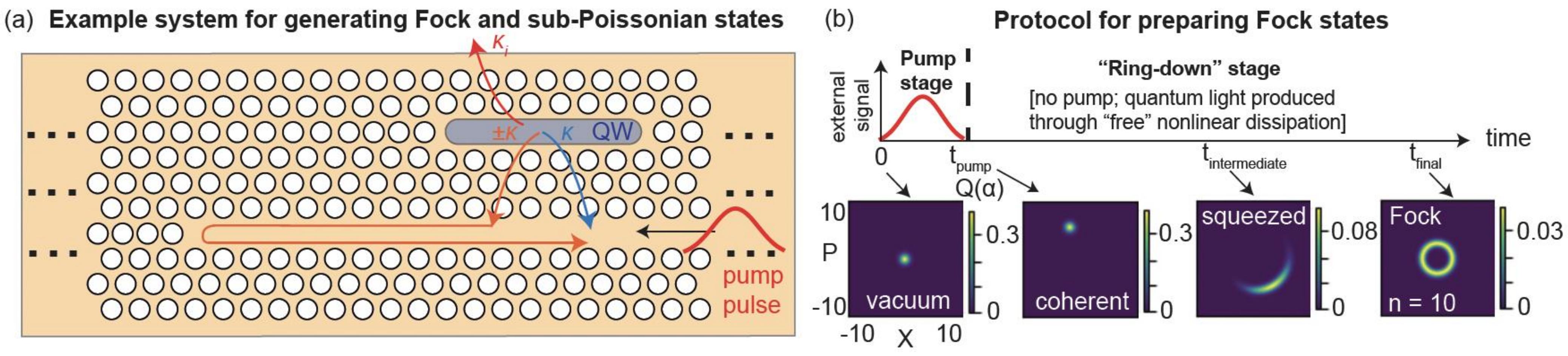


Quantum Kerr physics in exciton-polaritons
Imamoglu *et al.* Physical Review Letters (1997).
Bramati *et al.* Nature Communications (2014).
Fink *et al.* Nature Physics (2018).
Delteil & Fink *et al.* Nature Materials (2019).

How close are the implementations?

- Kerr nonlinearity within factor of 10 of loss (single-photon nonlinear regime)
- Exciton-polaritons even recently interfaced with gratings with BICs (Ardizzone *et al.* Nature (2022)).

Prospects for observation in systems of highly nonlinear exciton polaritons



Rivera* & Sloan* *et al.* arXiv:2211.01514

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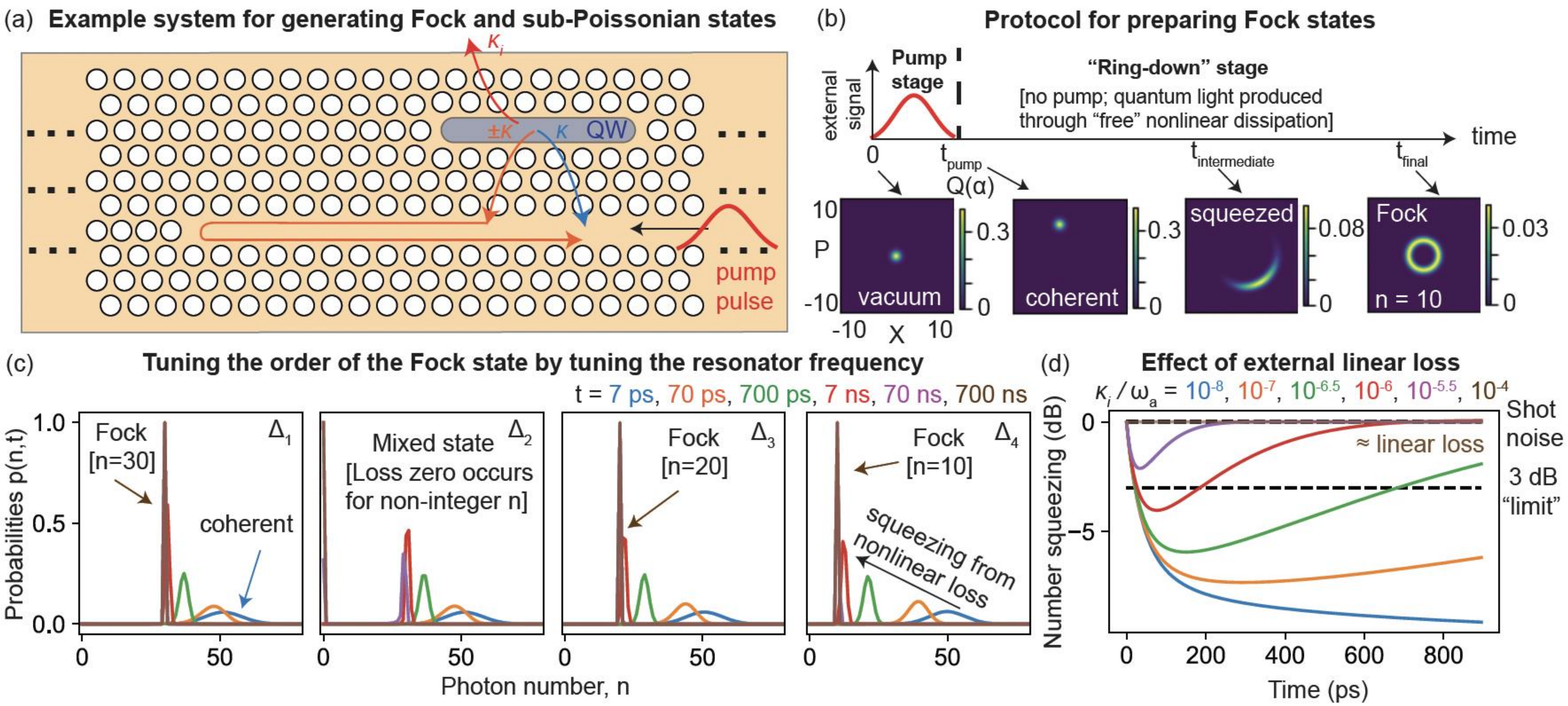
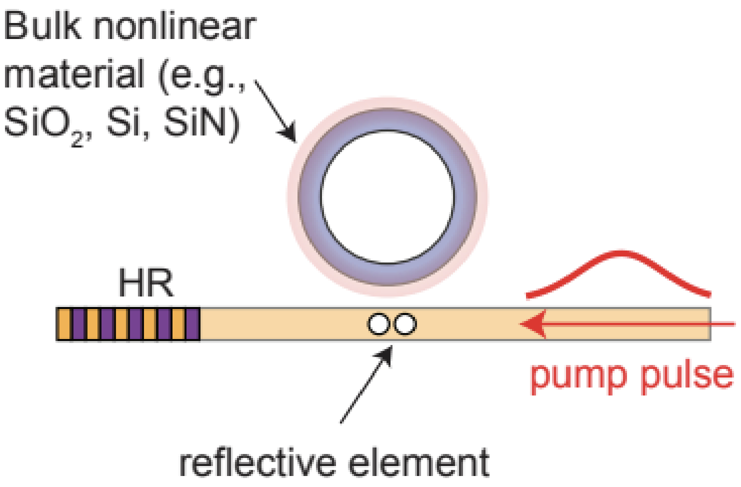
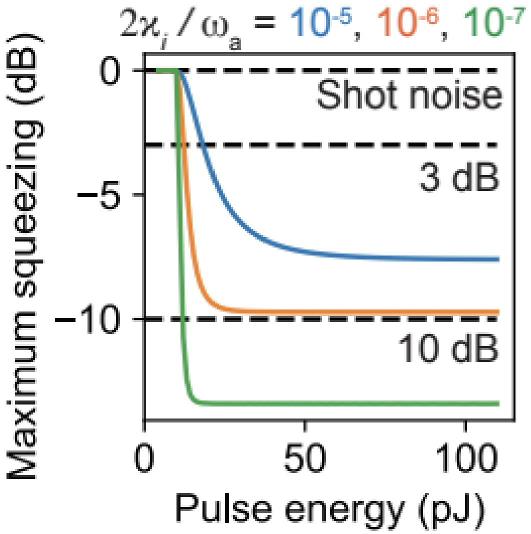
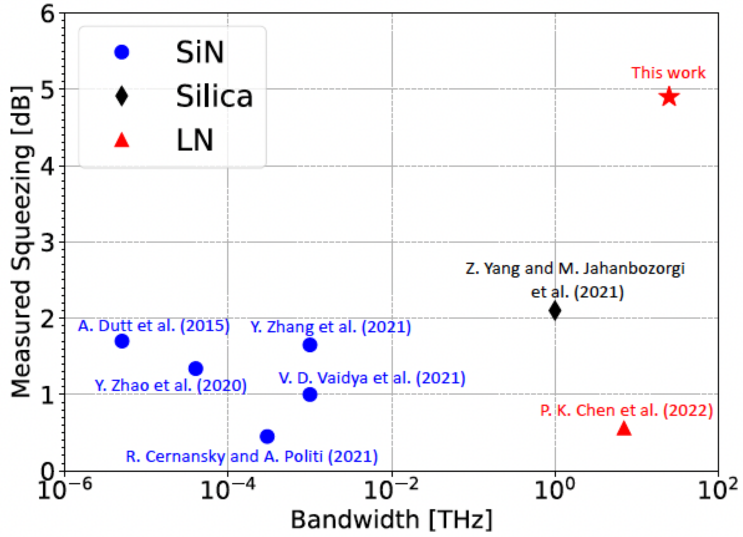


Rivera* & Sloan* *et al.* arXiv:2211.01514

Large squeezing in bulk nonlinear nanophotonic systems







Nehra *et al.* Science (2022).

- These effects also carry over to bulk nonlinear materials (silica, silicon, SiN, etc.)
- 10 dB squeezing (90% noise below classical shot noise level), considered a “magic number” and elusive

Rivera* & Sloan* *et al.* arXiv:2211.01514

A general quantum theory of outcoupling in nonlinear structures with dispersive loss

- A universal framework to predict quantum dynamics of a resonator with frequency-dependent loss.

Hamiltonian:

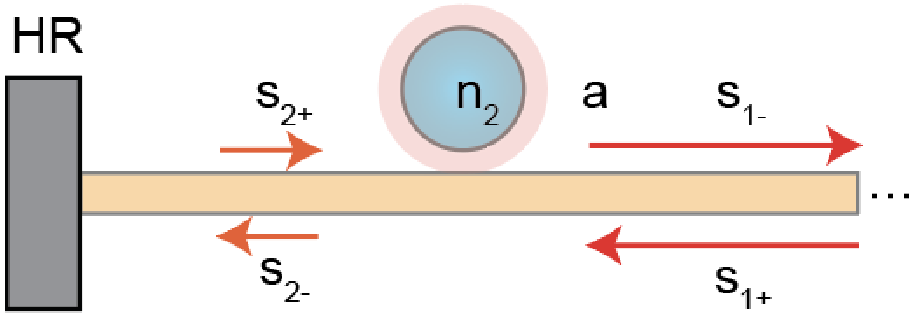
$$H/\hbar = \omega_a a^\dagger a + \beta\omega_a a^{\dagger 2} a^2 + \alpha(t)a + \alpha^*(t)a^\dagger$$
$$+ \sum_{i=1}^{N} \int \frac{d\omega}{2\pi}\, \omega s_i^\dagger(\omega) s_i(\omega)$$
$$+ \sum_{i=1}^{N} \int \frac{d\omega}{2\pi}\, i\left(K_{c,i}(\omega) s_i(\omega) a^\dagger - K_{c,i}^*(\omega) s_i^\dagger(\omega) a\right)$$

Dynamics of the photonic density matrix: $\dot{\rho} = -i[H_K + H_{\text{drive}}, \rho] + \mathcal{D}[\rho]$

Nonlinear dissipator:

$$\langle m|\mathcal{D}[\rho]|n\rangle = -\left(mK_l(\omega_{m,m-1}) + nK_l^*(\omega_{n,n-1})\right)\rho_{m,n} +$$
$$\sqrt{(m+1)(n+1)}\left(K_l(\omega_{m+1,m}) + K_l^*(\omega_{n+1,n})\right)\rho_{m+1,n+1}$$

From frequency-dependent lifetimes to nonlinear radiation loss: an example



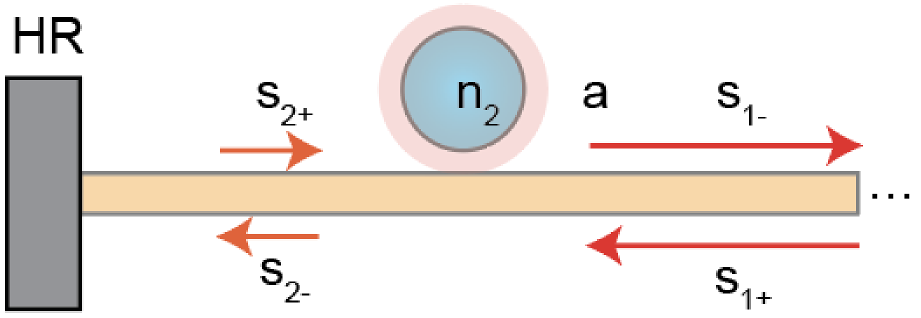
Heisenberg-TCMT with Kerr nonlinearity

$$\dot{a} = -i\omega_a a - (\kappa_1 + \kappa_2)a - i\omega_a \beta a^\dagger a^2 + \sqrt{2\kappa_1}e^{i\theta_1}s_{1+} + \sqrt{2\kappa_2}e^{i\theta_2}s_{2+}$$

Boundary conditions:

$$\begin{pmatrix} s_{1-} \\ s_{2-} \end{pmatrix} = C \begin{pmatrix} s_{1+} \\ s_{2+} \end{pmatrix} + \begin{pmatrix} \sqrt{2\kappa_1}e^{i\theta_1} \\ \sqrt{2\kappa_2}e^{i\theta_2} \end{pmatrix} a,$$ and $$s_{2+}(\omega) = -s_{2-}(\omega)e^{i\omega T}$$

From frequency-dependent lifetimes to nonlinear radiation loss: an example



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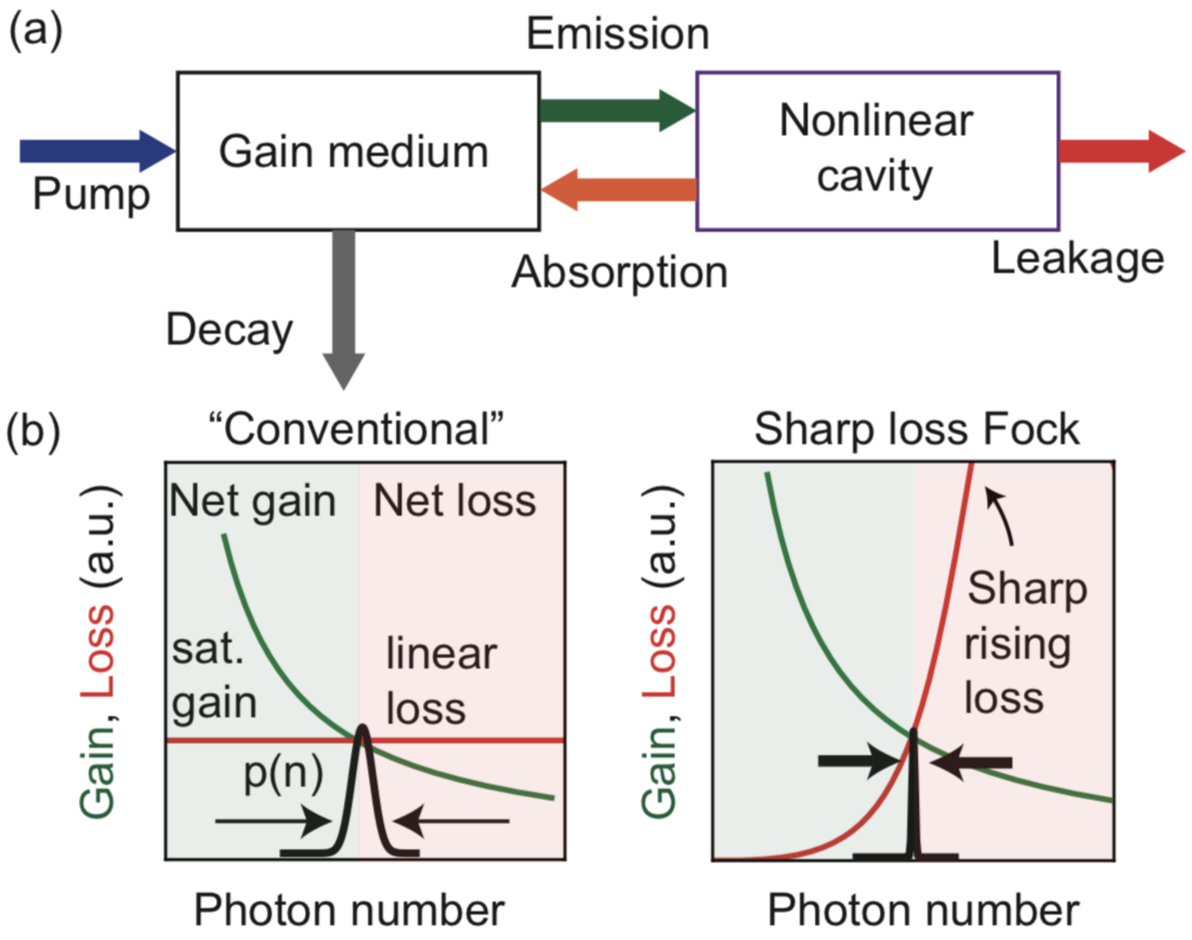
Non-Markovian equation for *a*

$$[i(\omega_a - \omega) + K_l(\omega)]\, a(\omega) = K_c(\omega)s_{1+}(\omega),$$

$$K_l(\omega) = 2\kappa\left(1 - \frac{e^{2i\theta_2}}{r_d - e^{-i\omega T}}\right) = 2\kappa\left(1 - \frac{r_d - it_d\sigma}{r_d - e^{-i\omega T}}\right)$$

$$K_c(\omega) = \sqrt{2\kappa}e^{i\theta_1}\left[1 + \frac{it_d e^{i(\theta_2 - \theta_1)}}{r_d - e^{-i\omega T}}\right].$$

Noise properties of lasers with nonlinear dissipation



- Presence of saturable gain adds terms to master eqn. (Lamb, Scully)

$$\dot{p}_n = G_n p_{n-1} - (L_n + G_{n+1}) p_n + L_{n+1} p_{n+1}.$$

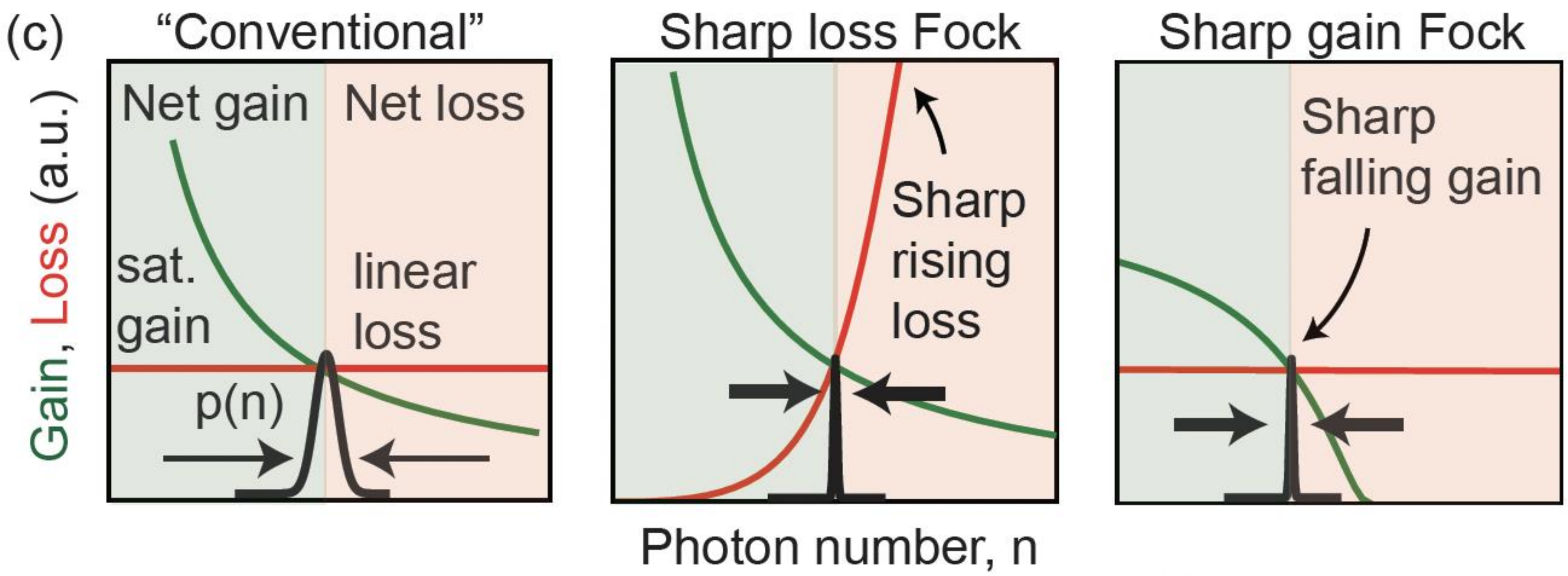
- Admits steady-state solution:

$$p_n = \frac{1}{Z} \prod_{m=1}^{n} \frac{G_m}{L_m}$$

- Noise given by

$$\Delta n = \left(1 - G_{\langle n \rangle +1} / L_{\langle n \rangle +1}\right)^{-1/2}$$

This nonlinear interaction enables a new kind of laser emitting Fock states



$$p(n) \approx \frac{1}{Z} \exp\left[-\frac{1}{2}|r'(n)|(n-\bar{n})^2\right] \implies \Delta n = -\left(\frac{d}{dn}\frac{G(n)}{\kappa(n)}\bigg|_{\bar{n}}\right)^{-1/2}$$

$$r(n) = G(n)/\kappa(n)$$