Predicting and controlling classical & quantum noise in multimode nonlinear photonics

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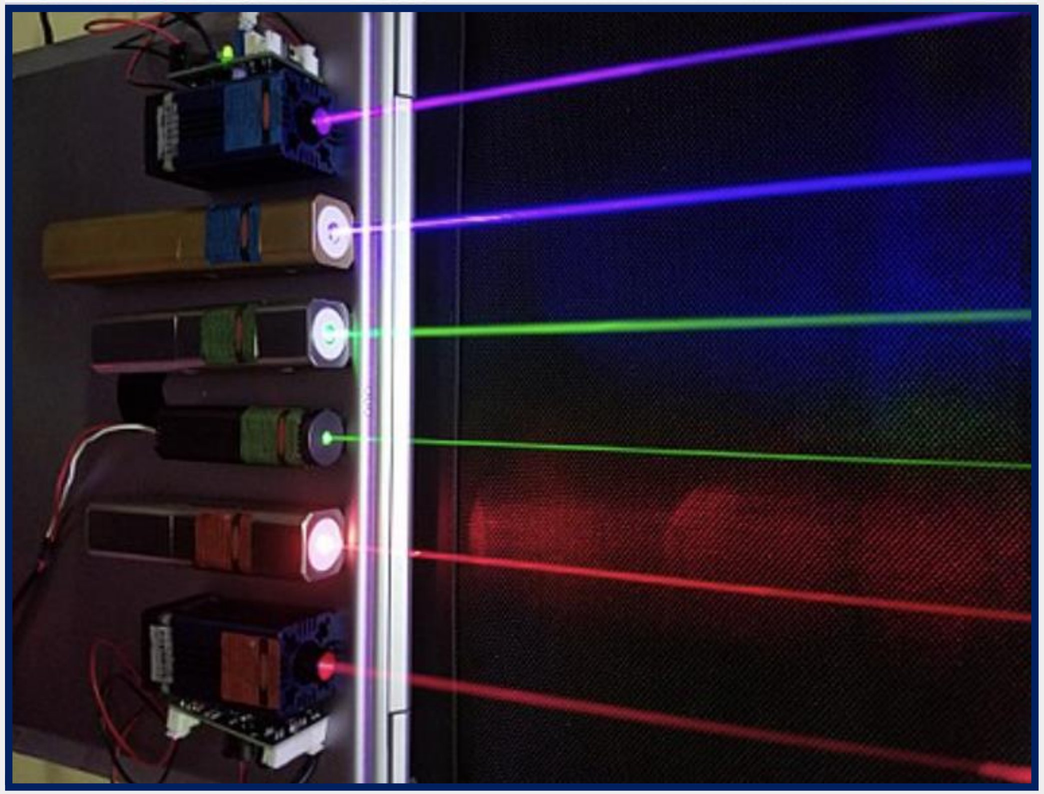
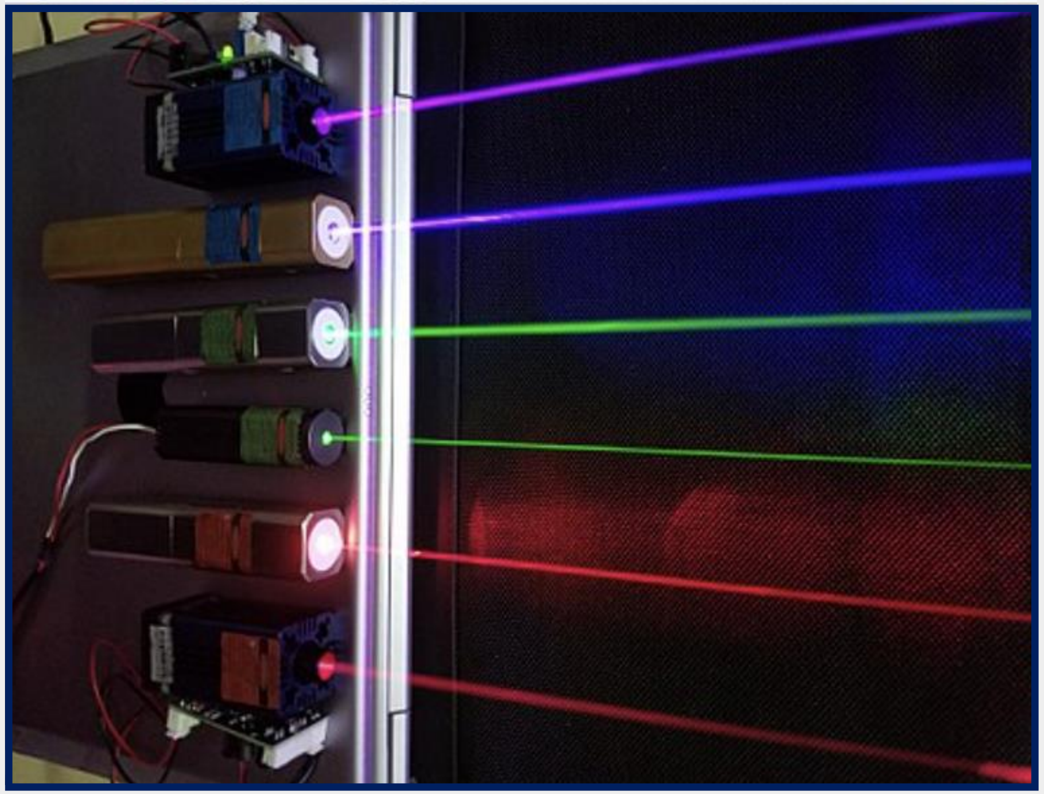
Lasers and their fluctuations



Source: Wikipedia

Lasers and their fluctuations



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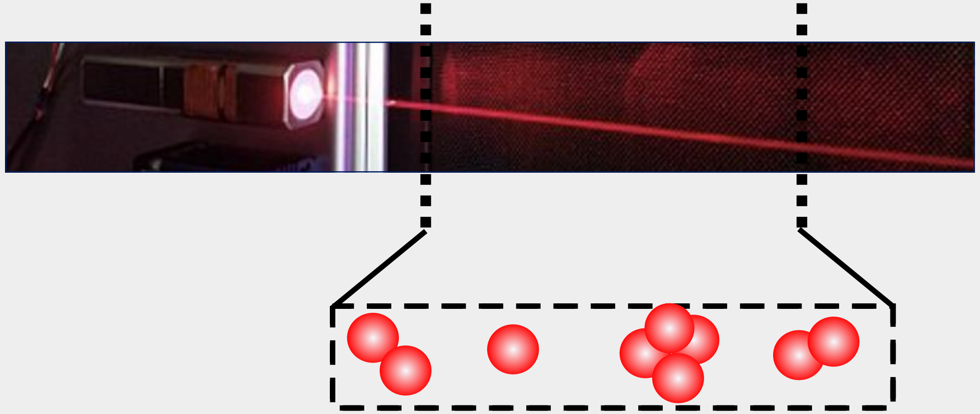
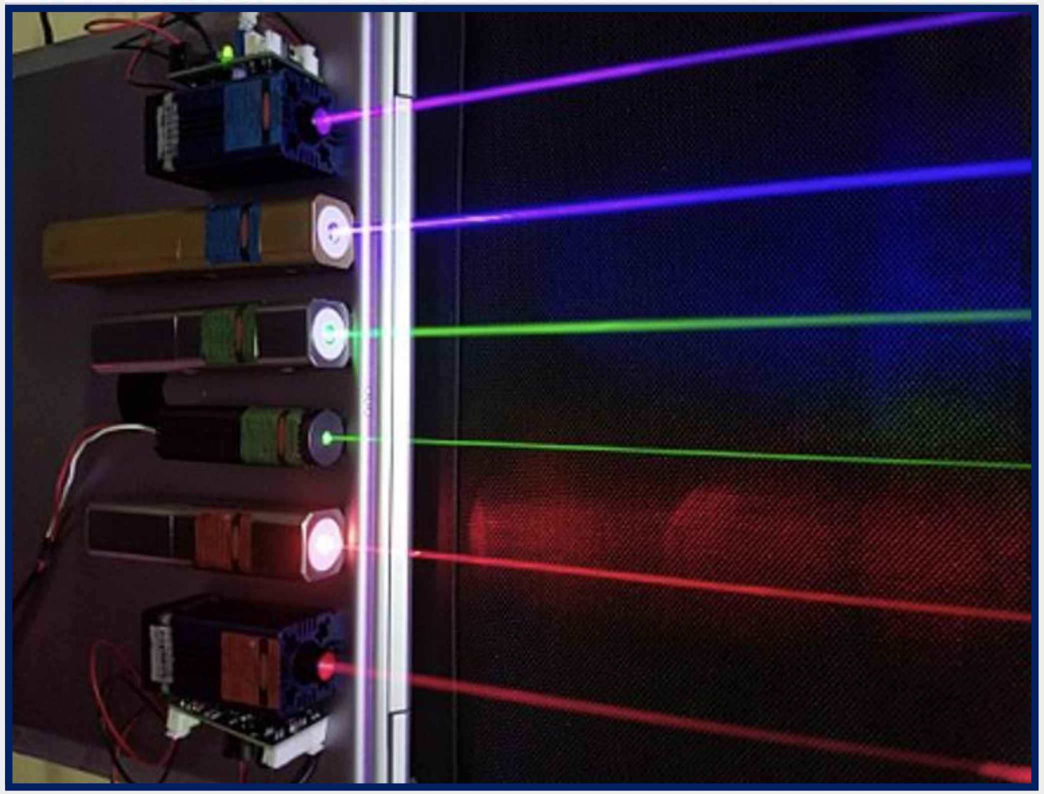


A beam as a collection of photons of fixed energy E = hf.

Lasers and their fluctuations



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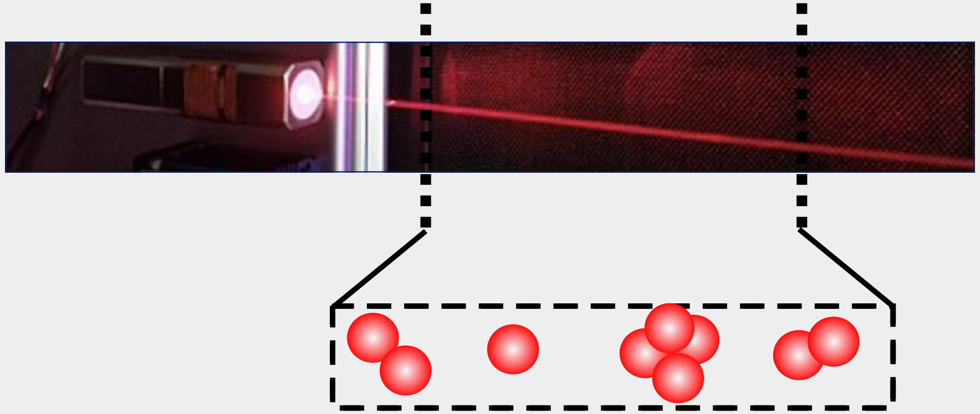
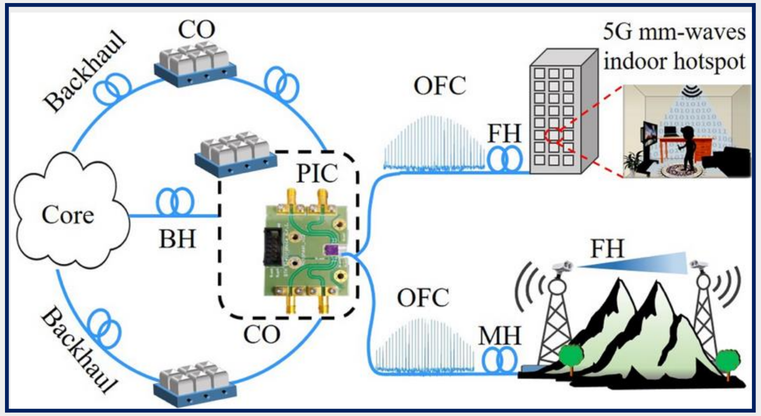


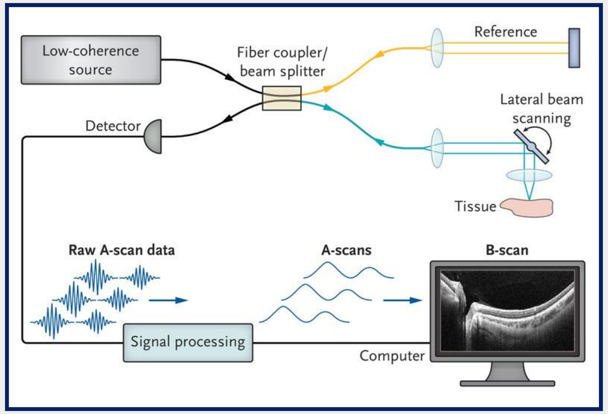
A beam as a collection of photons of fixed energy E = hf.

But also: phase, center frequency, arrival time, direction, position.

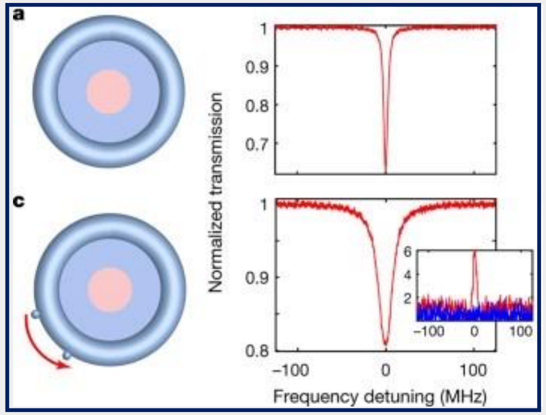
Systems limited by laser noise



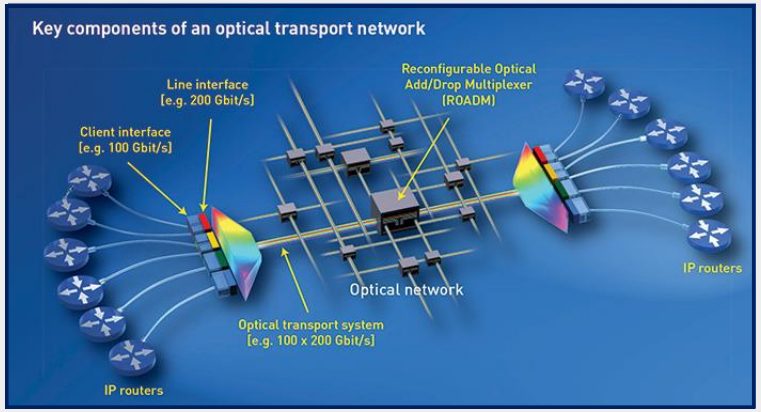
5G/6G networks



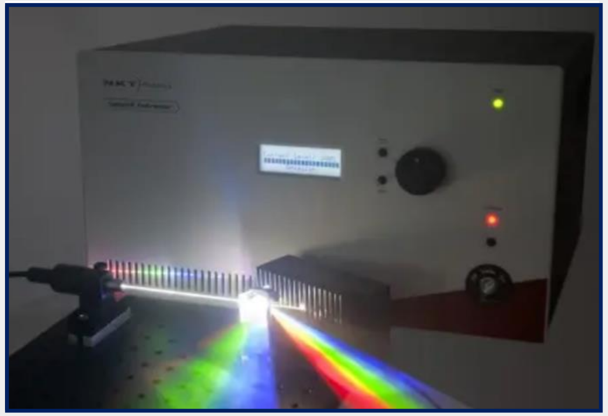
OCT



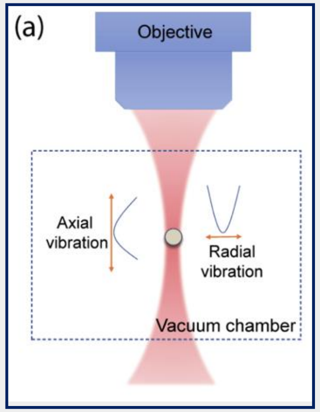
Biosensing



Optical fiber networks



Broadband spectroscopy



Atom traps

and *many* others!

Image credits: Nature, Optica, NEJM, Wiki, RP Photonics

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Noise in multimode nonlinear optics

Nonlinearity as a noise source

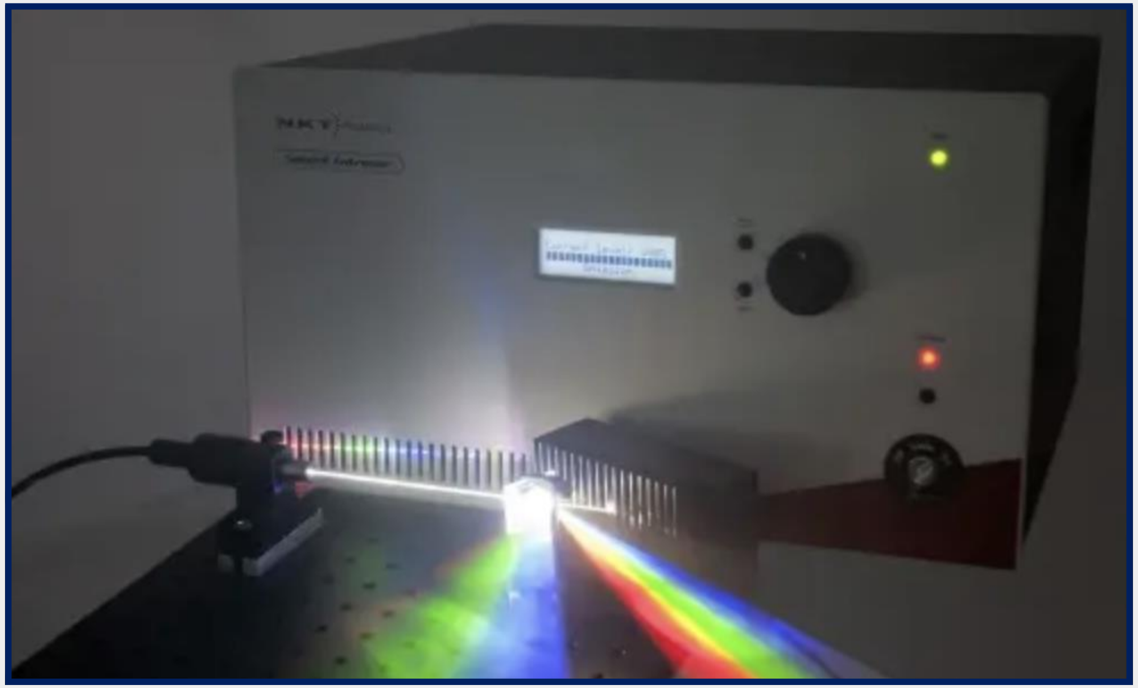

Image credit: NKT Photonics.

Many nonlinear phenomena amplify small changes in initial conditions (even quantum noise) such that the output noise can become significant compared to the mean field.

- Supercontinuum, transverse mode instability, modulation instability.
- **Hard to distill, expensive to predict.**

Nonlinearity as a noise reducer

On the other hand, we know from quantum optics that nonlinearity (second or third-order) can *induce squeezing,* noise below the level set by vacuum fluctuations – providing greater sensitivity.

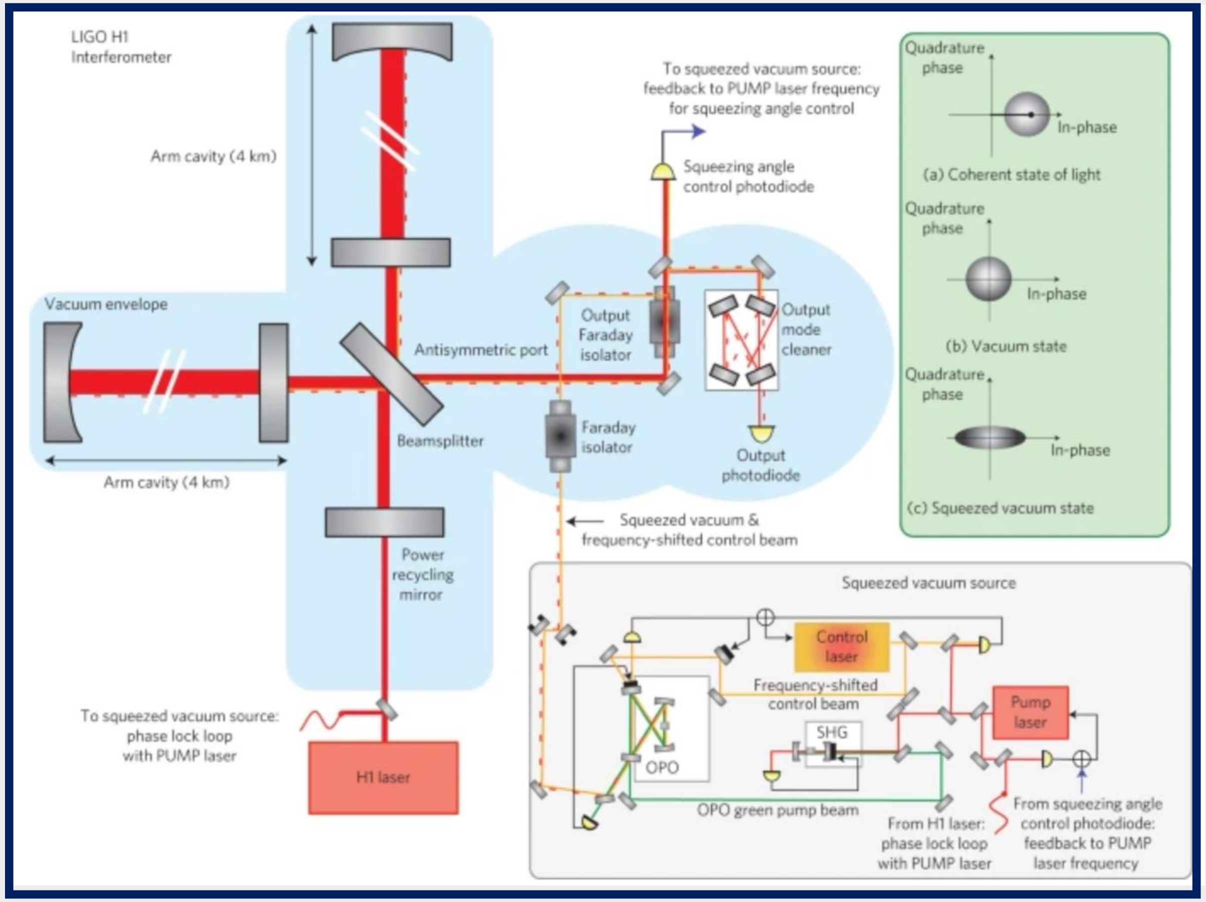

Image credit: Nature Physics (2013)

Noise in multimode nonlinear systems

- Is this picture relevant for complex sources? High-intensity, pulsed, spatially multimode systems with gain/loss/collective excitations?
  - The main issue is complexity. Generic nonlinearities amplify noise in ways that are hard to predict and intuit.

Noise in multimode nonlinear systems

- Is this picture relevant for complex sources? High-intensity, pulsed, spatially multimode systems with gain/loss/collective excitations?
  - The main issue is complexity. Generic nonlinearities amplify noise in ways that are hard to predict and intuit.
- The main questions which motivate this talk:
  - **(1) How can we efficiently identify physical mechanisms of noise generation and suppression in highly multimode nonlinear systems?**
  - **(2) Can nonlinearity be steered to reduce noise while maintaining mean-field?**
  - **(3) Can we do better than what linear and single-mode noise limits suggest?**

Noise in multimode nonlinear optics

Quantum noise in ultrafast pulses and lasers (frequency combs, supercontinuum, squeezing)

Bergman and Haus, *Opt. Lett.* 16.9 (1991): 663-665
Drummond *et al. Nature* 365.6444 (1993): 307-313
Friberg *et al. Phys. Rev. Lett.* 77, no. 18 (1996): 3775.
Corwin *et al. Phys. Rev. Lett.* 90, no. 11 (2003): 113904
Corney *et al. Phys. Rev. A* (2008): 023831
Rao *et al. Light: Sci. & Appl.* 10, no. 1 (2021): 133.
Nehra *et al. Science* 377.6612 (2022): 1333-1337.
Guidry et al. *Nat. Photon.* 16.1 (2022): 52-58.
Ng *et al. arXiv:2307.05464* (2023).
Herman *et al. Science* 387, no. 6734 (2025): 653-658.
Lustig *et al. Nat. Photon.* (2025): 1-8
Jia *et al. Nature* 639.8054 (2025): 329-336

Spatially multimode noise

Boyer *et al. Science* 321.5888 (2008): 544-547
Ohad *et al. Nature Physics* 18.9 (2022): 986-993
Wisal et al. *Phys. Rev. X* 14, no. 3 (2024): 031053.
Rothe *et al. Science* 390.6769 (2025): 173-177.
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Where we are operating:
- Tools for unpacking mechanisms of noise generation and suppression in *complex* systems
- Tools for experimental control techniques for complex systems
- Scaling up intensity of quantum resources.

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Quantum sensitivity analysis (QSA): an efficient approach for multimode noise

Zia Uddin* and **Rivera*** *et al.* Nature Photonics (2025).

Quantum noise in nonlinear optics

General nonlinear map (MM-GNLSE, LLE, HHG, etc.):

$$\underbrace{\partial_t \alpha_i}_{\text{complex field amplitudes}} = \underbrace{f_i(\{\boldsymbol{\alpha}\})}_{\text{nonlinear dynamics}} + \underbrace{\Gamma_i}_{\text{noise forces}}$$

Noise sources: initial conditions (vacuum fluctuations, technical noise, squeezing), gain and loss, other excitations (e.g., phonons)).

How can we predict the ways noise is amplified and de-amplifed by nonlinear dynamics, in very multimode nonlinear systems?

QSA: a factorization formula for noise

Consider an observable $X[t, \{\boldsymbol{\alpha}(0), \boldsymbol{\alpha}^*(0), \boldsymbol{\Gamma}, \boldsymbol{\Gamma}^*\}]$ which depends on the initial conditions $\boldsymbol{\alpha}(0)$ and time-dependent noise sources $\boldsymbol{\Gamma}(\mathrm{t})$. *In the linearized approximation* (multimode Bogoliubov), the variance in X is given by:

$$(\Delta X(t))^2 = (\boldsymbol{\lambda}^X(0))^T \begin{pmatrix} \langle \delta\mathbf{a}(0)\delta\mathbf{a}(0)\rangle & \langle \delta\mathbf{a}(0)\delta\mathbf{a}^\dagger(0)\rangle \\ \langle \delta\mathbf{a}^\dagger(0)\delta\mathbf{a}(0)\rangle & \langle \delta\mathbf{a}^\dagger(0)\delta\mathbf{a}^\dagger(0)\rangle \end{pmatrix} \boldsymbol{\lambda}^X(0)$$
$$+ \int_0^t \int_0^t dt' dt'' \, (\boldsymbol{\lambda}^X(t'))^T \begin{pmatrix} \langle \boldsymbol{\Gamma}(t')\boldsymbol{\Gamma}(t'')\rangle & \langle \boldsymbol{\Gamma}(t')\boldsymbol{\Gamma}^\dagger(t'')\rangle \\ \langle \boldsymbol{\Gamma}^\dagger(t')\boldsymbol{\Gamma}(t'')\rangle & \langle \boldsymbol{\Gamma}^\dagger(t')\boldsymbol{\Gamma}^\dagger(t'')\rangle \end{pmatrix} \boldsymbol{\lambda}^X(t'')$$

with: $\boldsymbol{\lambda}^X(t') = \left(\frac{\partial X(t)}{\partial \alpha(t')}, \frac{\partial X(t)}{\partial \alpha^*(t')} \right)^T$ being a classical gradient.

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From QSA to computational efficiency

$\lambda_i^X(t)$ $(i = 1, \ldots, M)$ is an *adjoint*: satisfying an auxiliary *linear* ODE:

$$\partial_t \lambda_i^X = -\sum_{j=1}^{M} \left(\frac{\partial f_j}{\partial A_i} \lambda_j^X + \frac{\partial f_j^*}{\partial A_i} \lambda_j^{X*} \right)$$

$$\partial_t \alpha_i = f_i(\{\boldsymbol{\alpha}\}) + \Gamma_i$$

Computational complexity is one forward solve & one backward solve

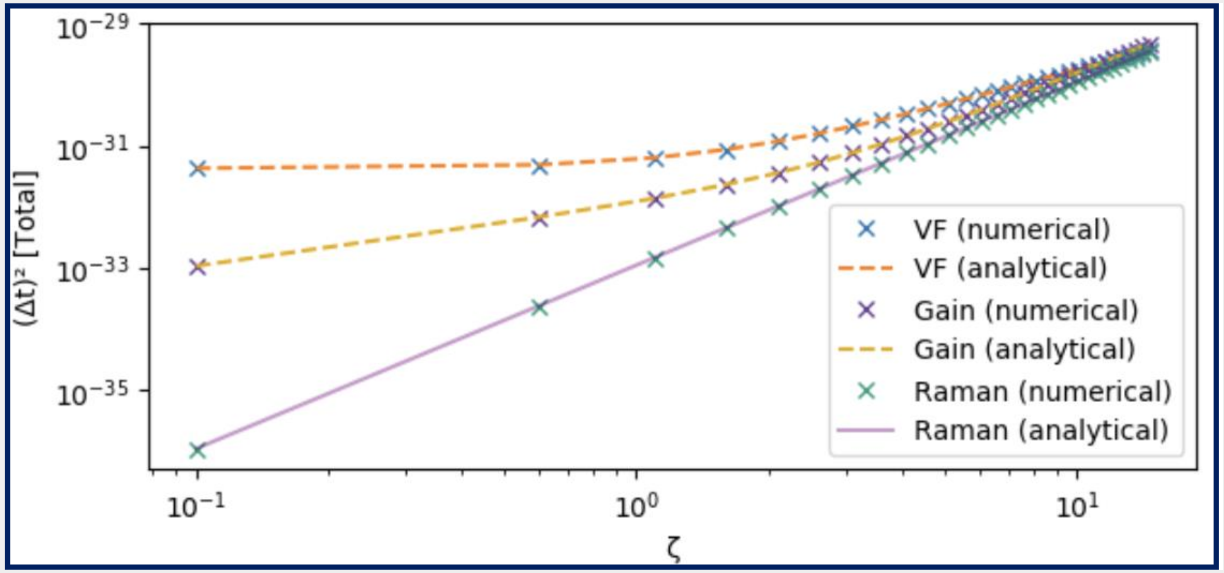
- *Much lower* than typical approaches
  - Cumulant EOM (2M), stochastics (k >> 1 forward solves)
- Different noise distributions are analytical by QSA factorization.
- Gradients → dynamical systems, optimization, inverse design…

Example: timing jitter of solitons

Soliton jitter due to vacuum fluctuations, amplifier noise, and Raman noise:

$$(\Delta t)^2 = \underbrace{\left(\frac{\pi^2}{12\langle n\rangle} + \frac{1}{3\langle n\rangle}\zeta^2\right)}_{\text{vacuum fluctuations}} + g\underbrace{\left(\frac{\pi^2}{6\langle n\rangle}\zeta + \frac{2}{9\langle n\rangle}\zeta^3\right)}_{\text{amplifier noise}} + \underbrace{\left(\frac{16F_R(0)}{45\langle n\rangle}\zeta^3\right)}_{\text{Raman}}$$

$$\zeta = \frac{z}{L_D}$$



(This and other examples can be found and played with in a repo linked at the end of the talk.)

https://github.com/nrivera494/multimode-noise-modeling

Credit to Michael Horodynski and Fadi Farook.

What we have been doing with QSA

- Sensitivity analysis and dimensionality reduction of complex noise dynamics in femtosecond pulse propagation:
  - *Zia Uddin* and **Rivera*** et al. Nature Photonics (2025).*

- Optimization of noise; co-control of mean-fields and fluctuations in spatiotemporally multimode systems:
  - *Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482* [submitted].

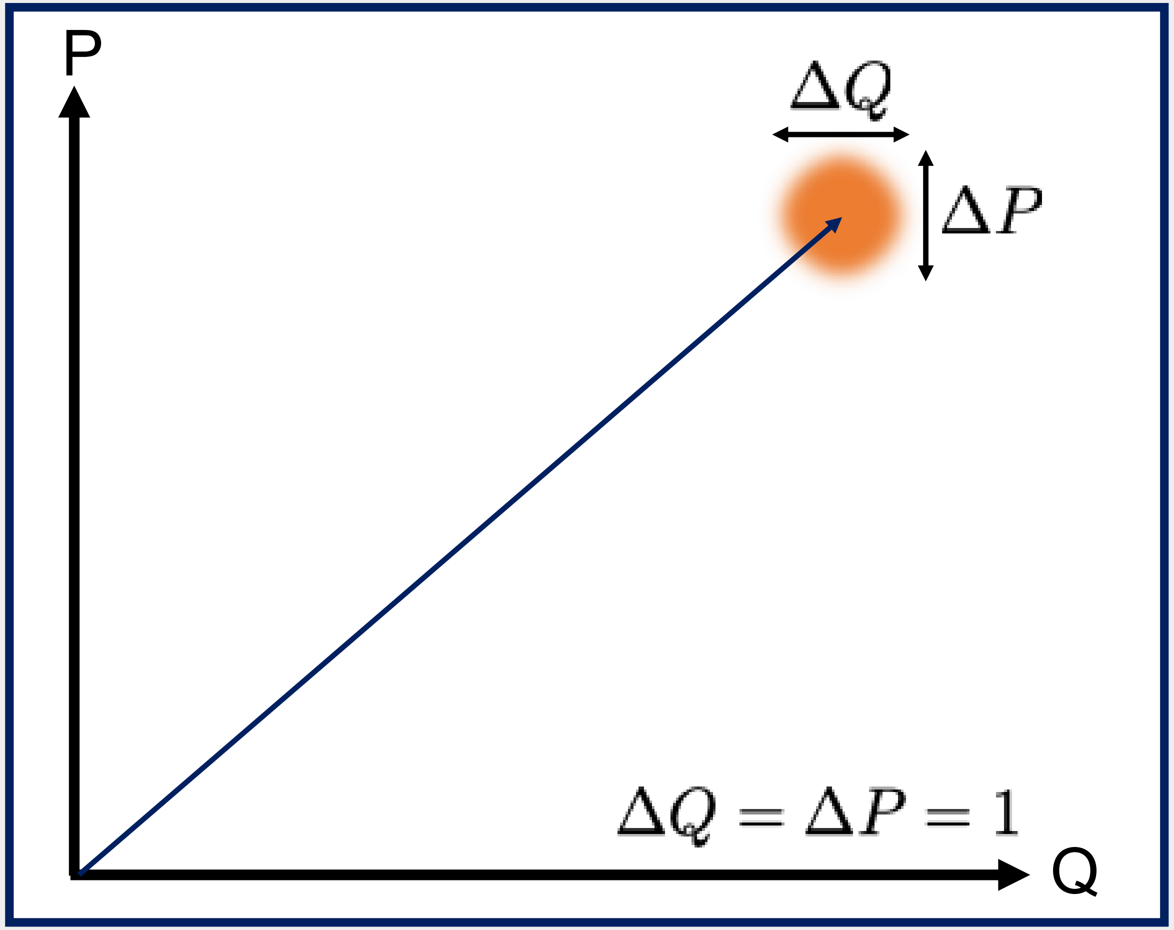
- Identifying optimal Gaussian resources (multimode squeezed states) for a given application:
  - ***Rivera** et al. Nanophotonics (2025).*

Shiekh Zia Uddin

Noise-immune quantum correlations of intense light

Zia Uddin* and **Rivera*** *et al.* Nature Photonics (2025).

Lasers as coherent states

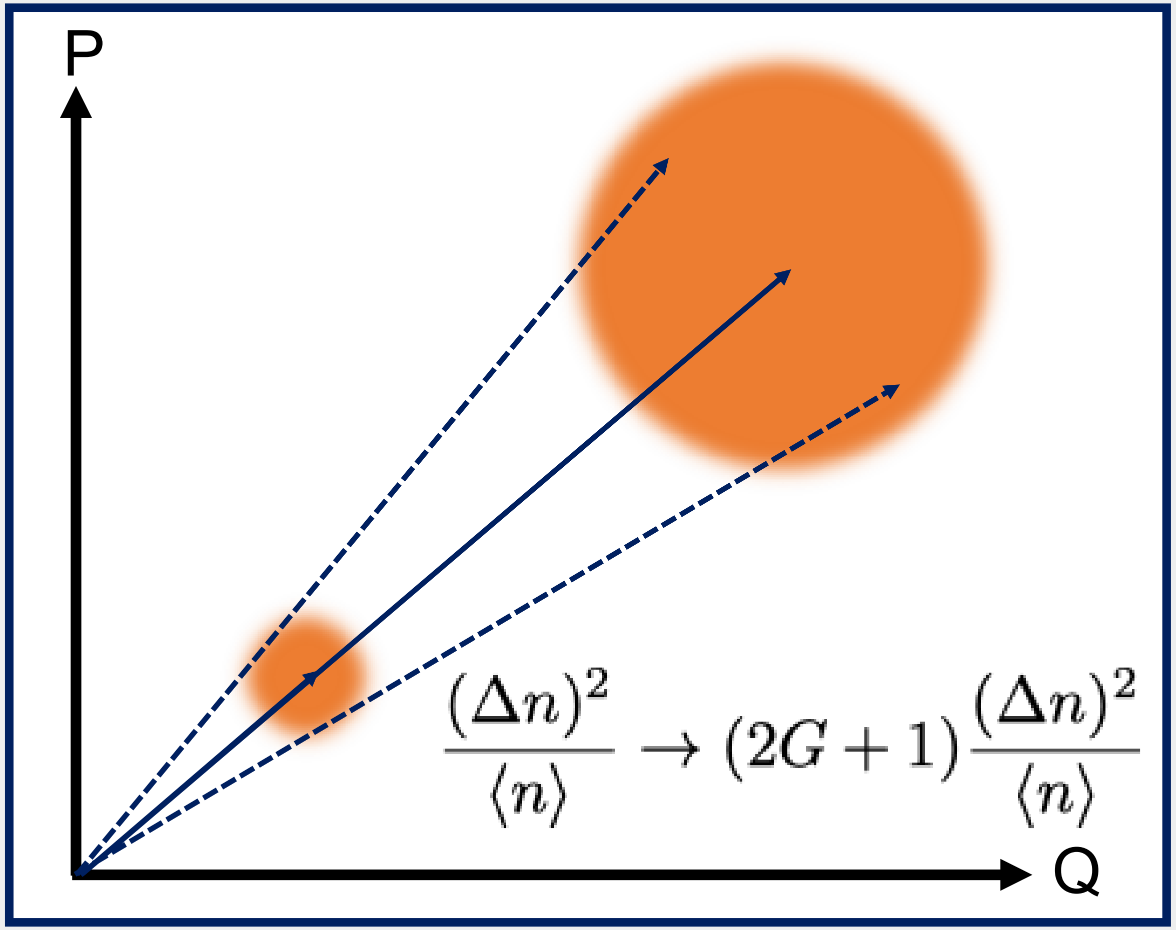


We can see the coherent state as something classical plus zero-point fluctuations of the vacuum state.

$$a = \underbrace{\langle a \rangle}_{\text{mean}} + \underbrace{\delta a}_{\text{noise}}$$

mean + noise

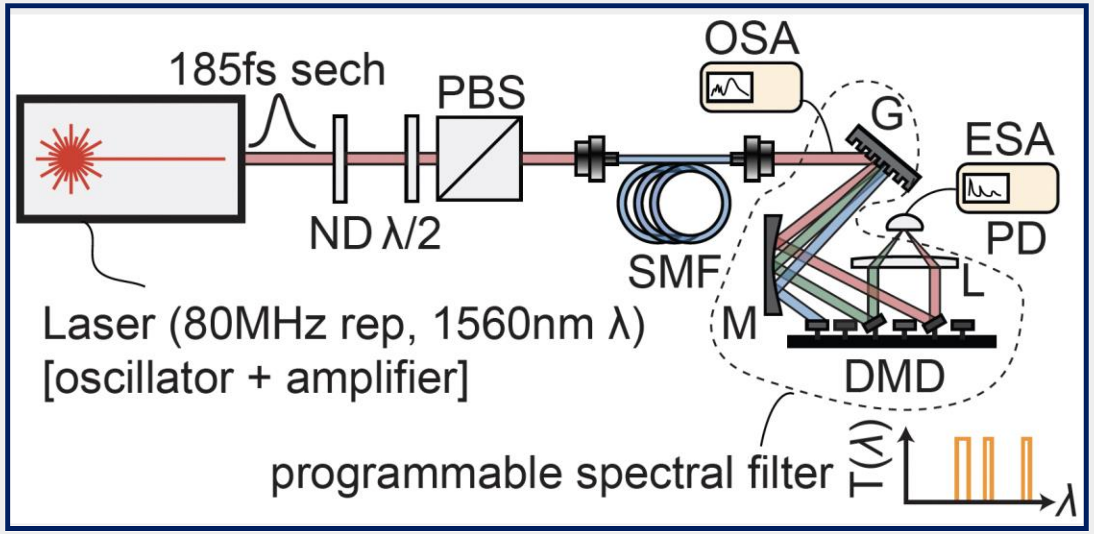
The reality of many light sources



$$\frac{(\Delta n)^2}{\langle n \rangle} \rightarrow (2G+1)\frac{(\Delta n)^2}{\langle n \rangle}$$

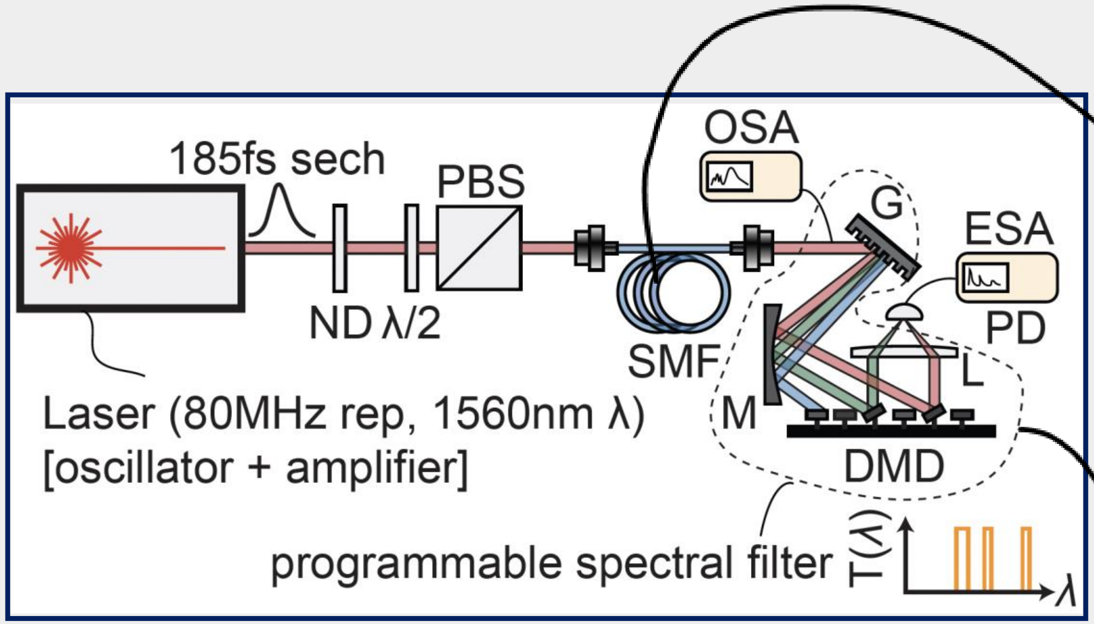
Now, if we shrink one quadrature's fluctuations by some factor, it can still be above the zero-point level.

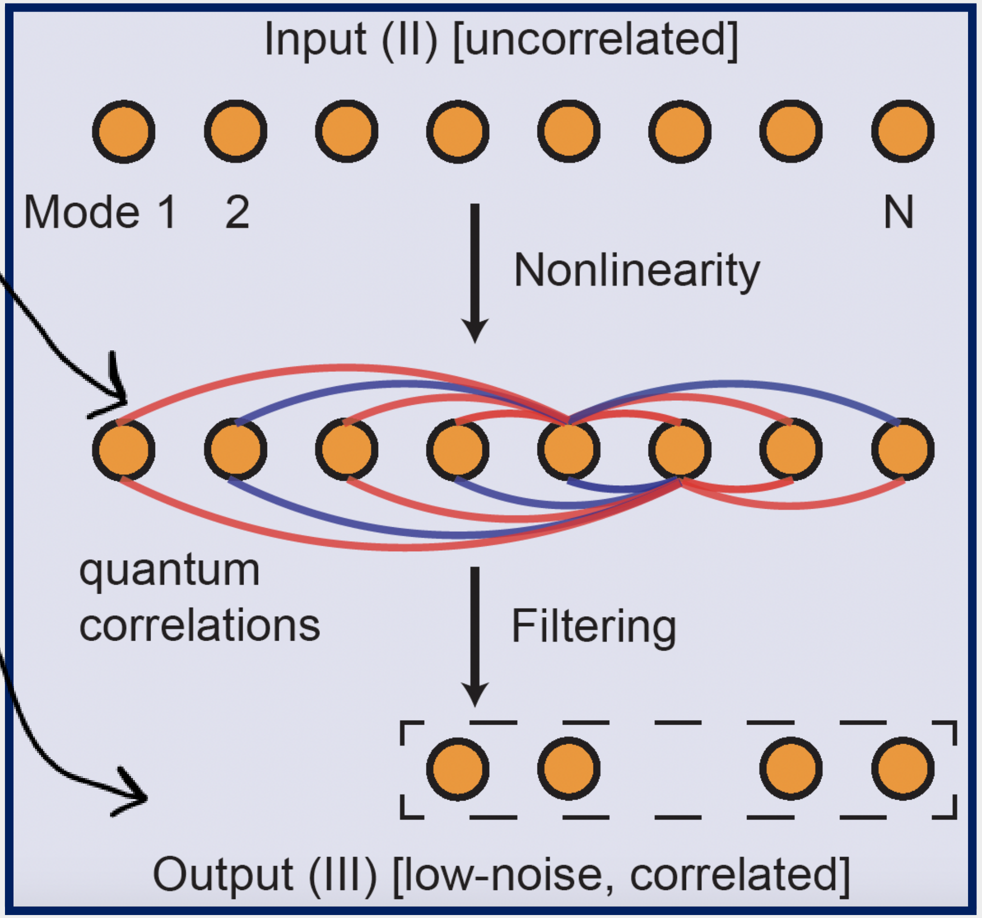
Squeezing of noisy pulses in fiber



S. Z. Uddin*, **N. Rivera***, *et al. Nature Photonics (2025).*

Squeezing of noisy pulses in fiber





S. Z. Uddin*, **N. Rivera***, *et al.* *Nature Photonics (2025).*

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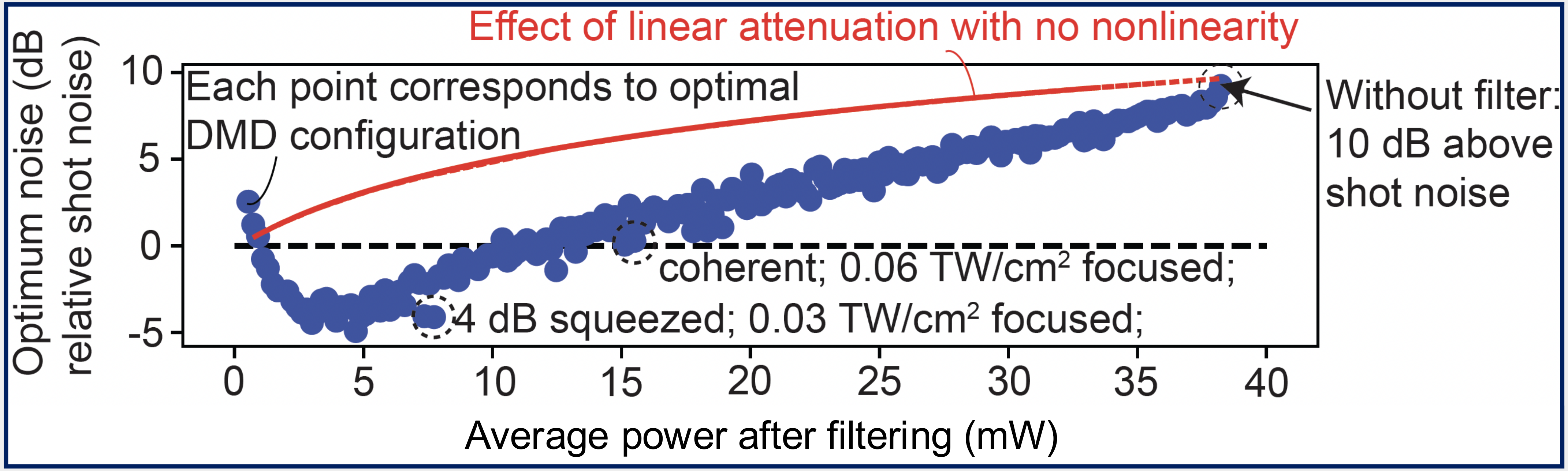
Squeezing of noisy pulses in fiber



The correlations from nonlinearity, plus filtering, allow much stronger noise reduction than what linear attenuation can give (here by almost 10x), and generate intense light with quantum noise.

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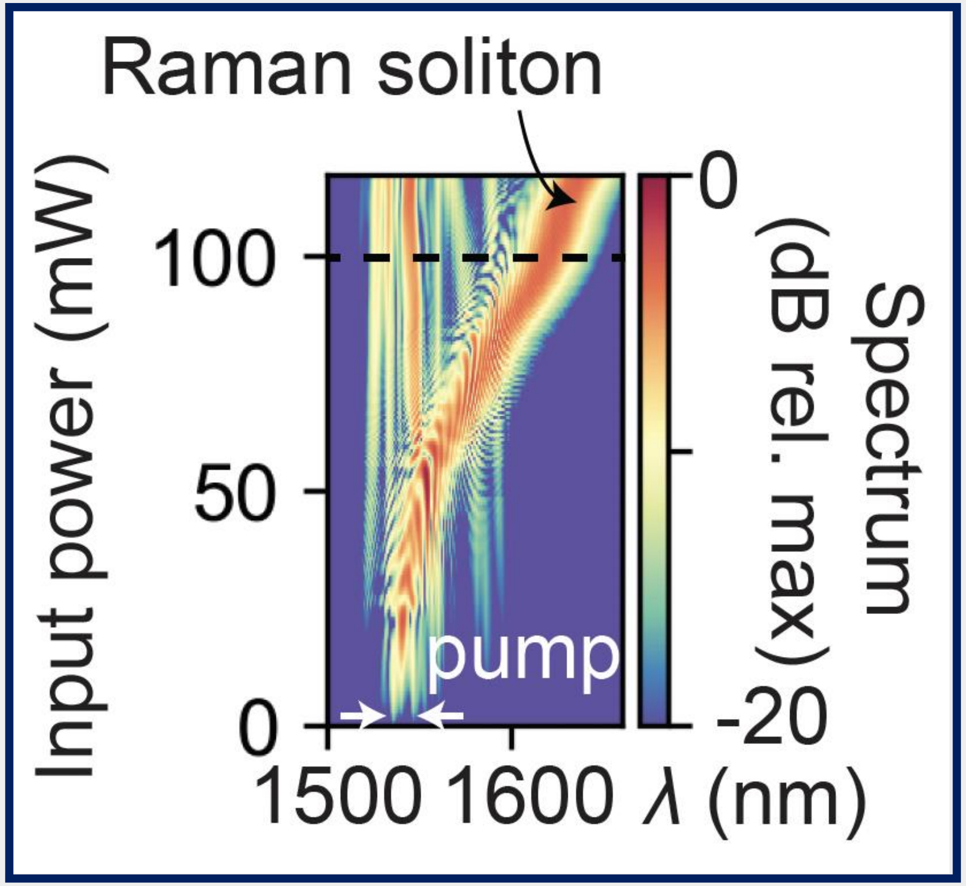
S. Z. Uddin*, **N. Rivera***, *et al.* *Nature Photonics (2025).*

Nonlinear dynamics in fibers



S. Z. Uddin*, **N. Rivera***, *et al. Nature Photonics (2025).*

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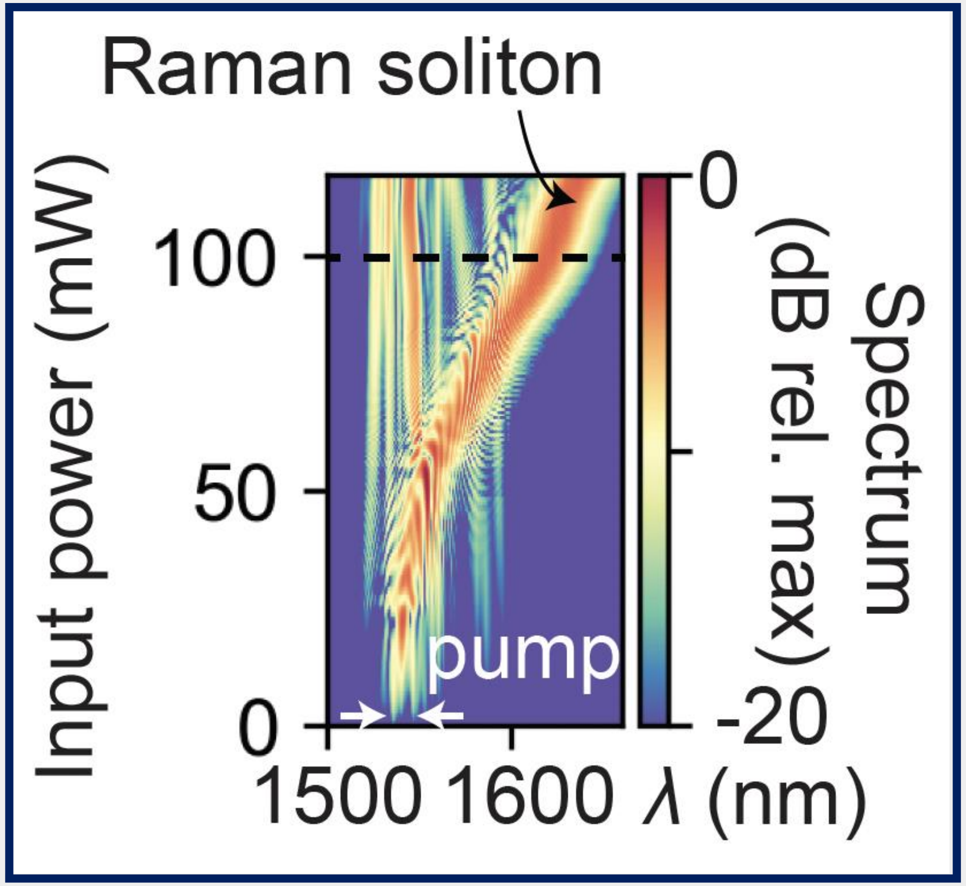
Nonlinear dynamics in fibers



Dispersion

$$k = k(\omega)$$

Four-wave mixing (Kerr)

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

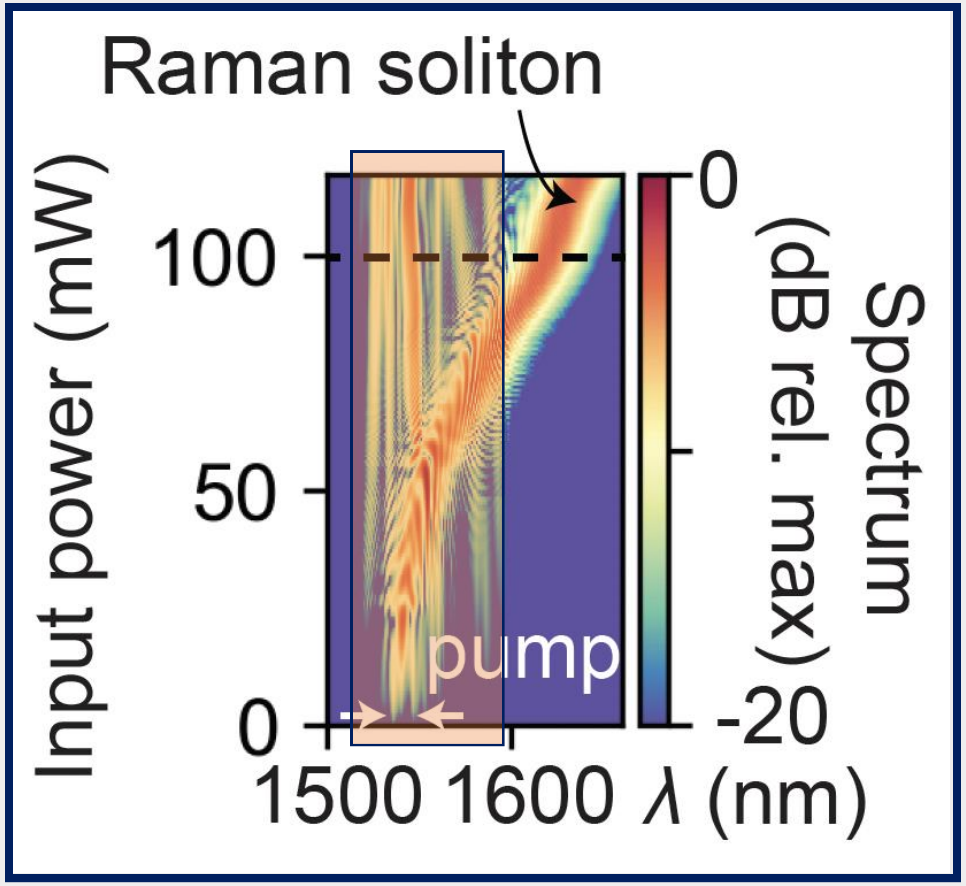
$$k(\omega_1) + k(\omega_2) = k(\omega_3) + k(\omega_4)$$

Raman scattering

$$\omega_1 = \omega_2 \pm \Omega$$

S. Z. Uddin*, **N. Rivera***, *et al.* *Nature Photonics (2025).*

Mechanism: noise-immune squeezing



Effect 1: noise decoupling

$$(\Delta X)^2 = \sum_{\omega' \in \omega_{\mathrm{amp}}} \left| \frac{\partial X}{\partial \alpha^{\mathrm{in}}_{\omega'}} \right|^2 F^{\mathrm{in}}_{\omega'} + \sum_{\omega' \notin \omega_{\mathrm{amp}}} \left| \frac{\partial X}{\partial \alpha^{\mathrm{in}}_{\omega'}} \right|^2$$

when pulse redshifts enough, its noise becomes less influenced by noise in the amplifier band.

Effect 2: optimal choice of X by choice of filter.

S. Z. Uddin*, **N. Rivera***, *et al. Nature Photonics (2025).*

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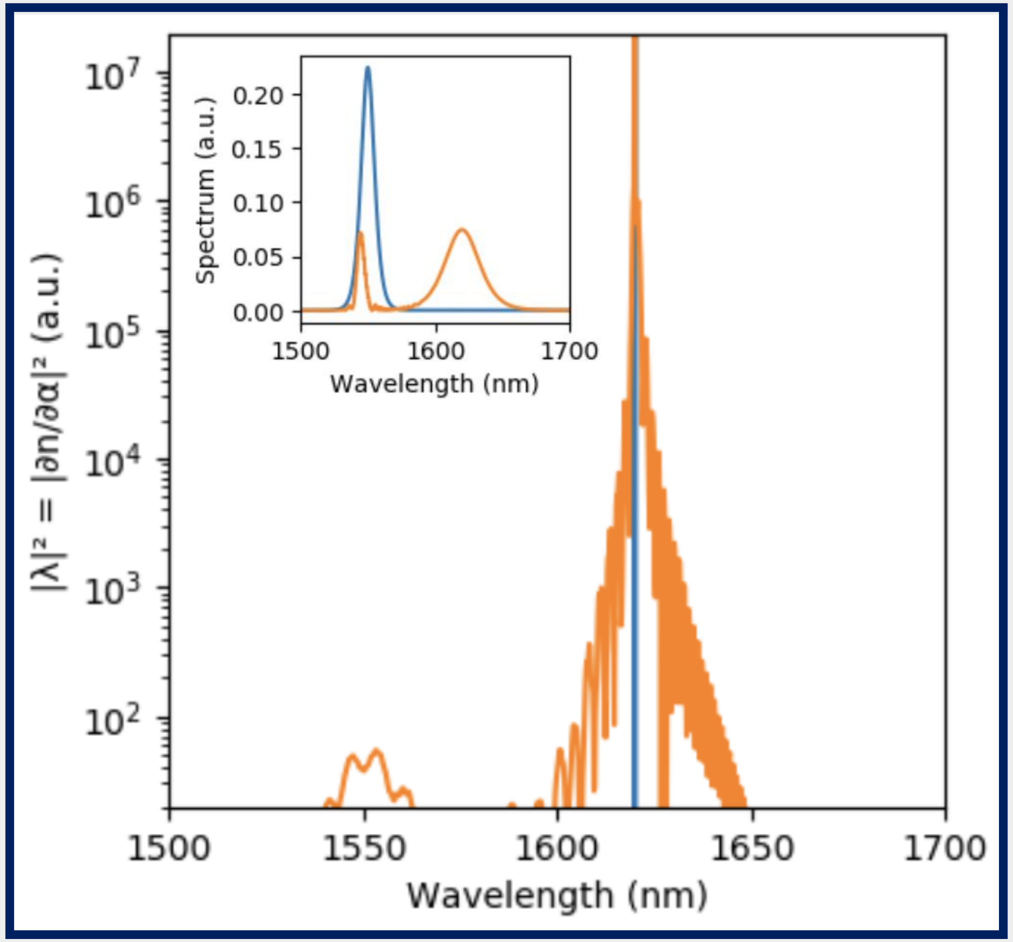
QSA of the Raman soliton peak intensity

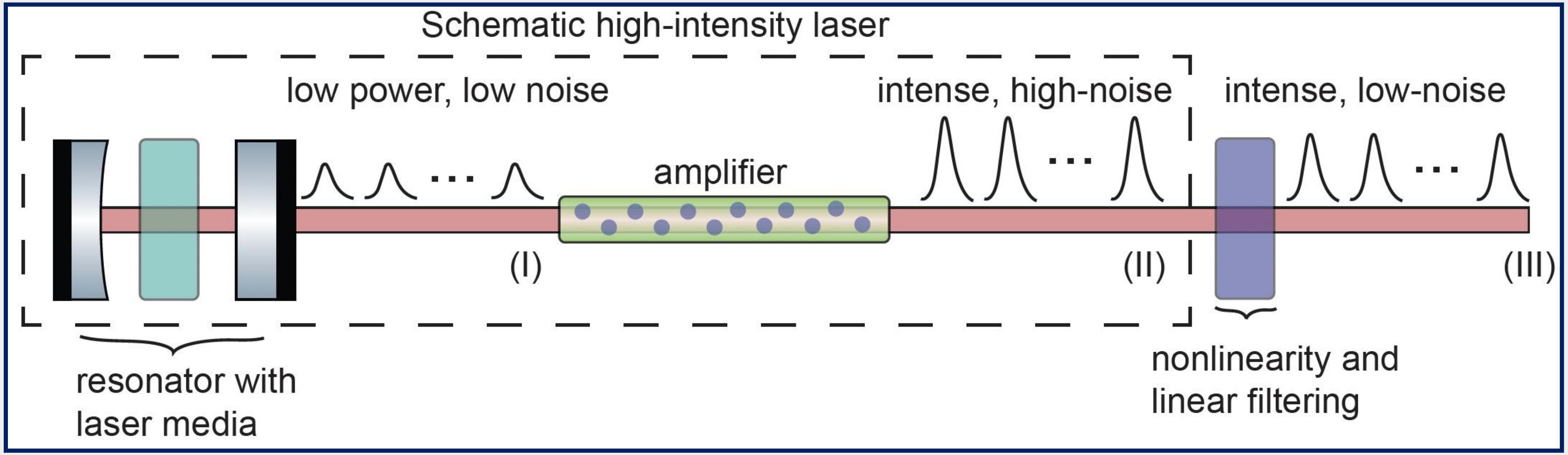


Raman spectral peak intensity is insensitive to input pulse fluctuations: only cares about vacuum fluctuations at Raman soliton wavelength. Squeezing filters have same property.

Adding 100x excess noise to the pump: only a 1% increase in intensity noise of the Raman peak (see more in repo linked at end of talk).

S. Z. Uddin*, **N. Rivera***, *et al.* *Nature Photonics (2025).*

Quantum nonlinear optical amplifiers



Upshot: being able to squeeze amplified light sources opens up the range of systems and sources that can be used to generate squeezed light.

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S. Z. Uddin*, **N. Rivera***, *et al. Nature Photonics (2025).*

Jamison Sloan

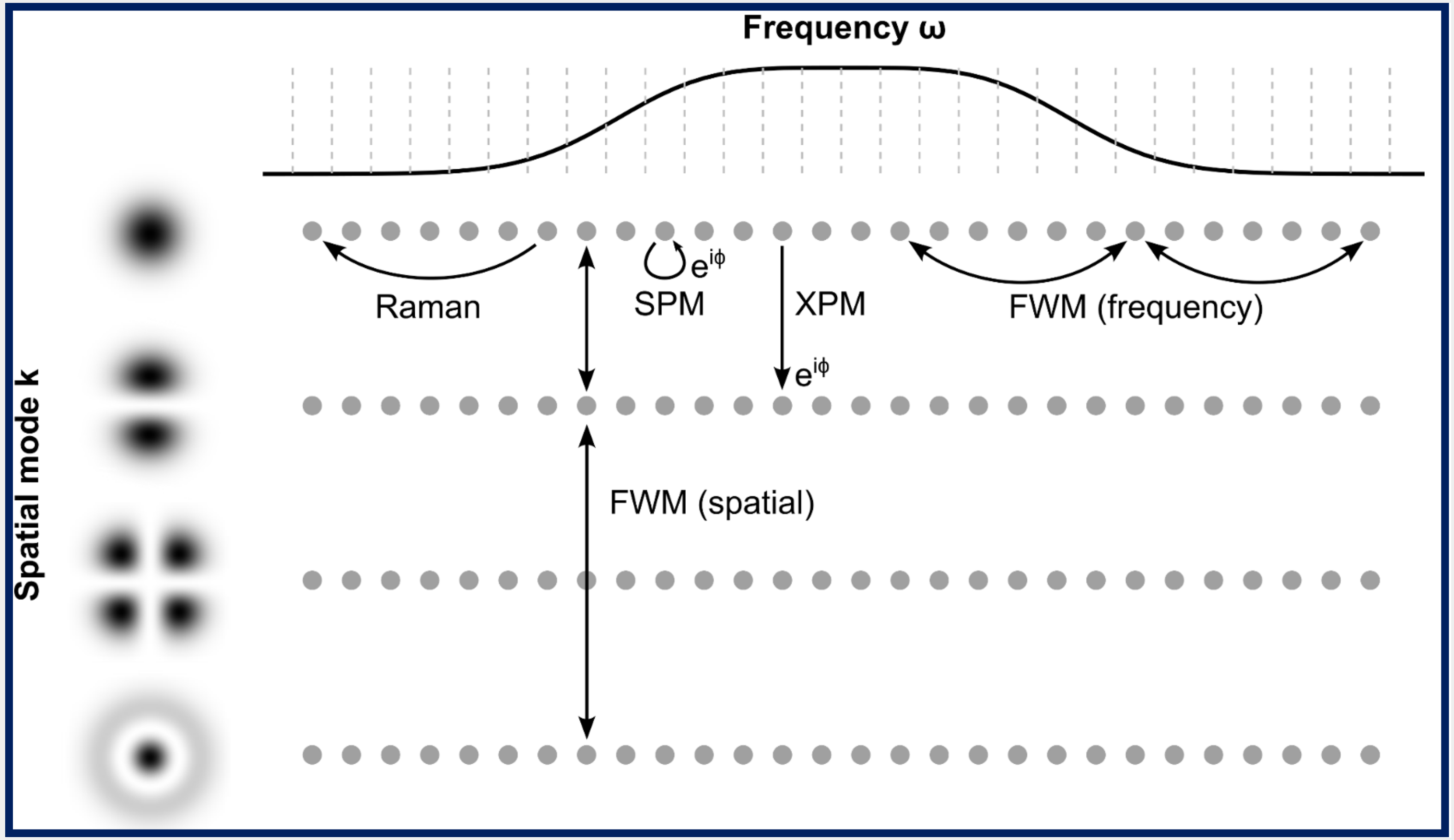
Michael Horodynski

Shiekh Zia Uddin

Programming the spatiotemporal quantum noise of light

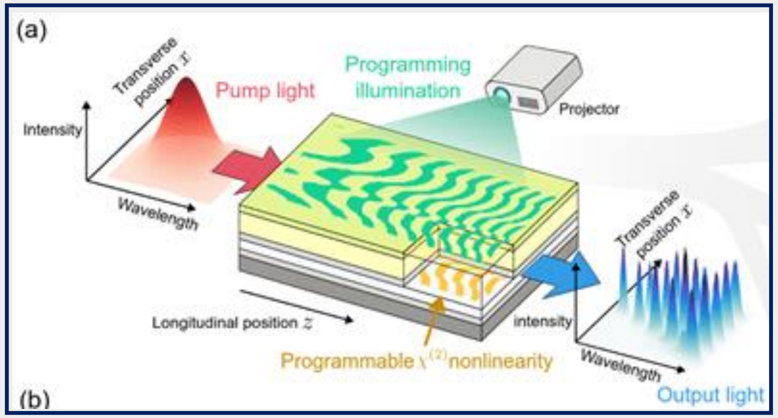
Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Spatiotemporal multimode nonlinearity

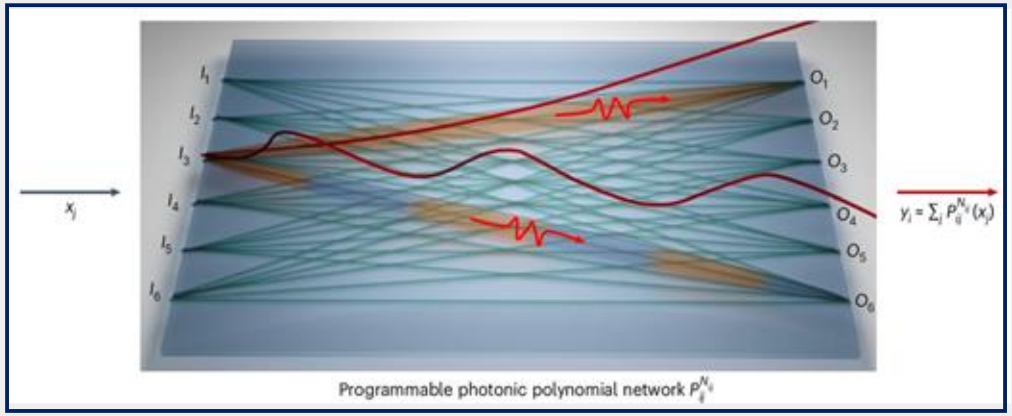


See work by groups of Christodoulides, Wise, Lipson, Cao, Rotter, Hafezi, and others.

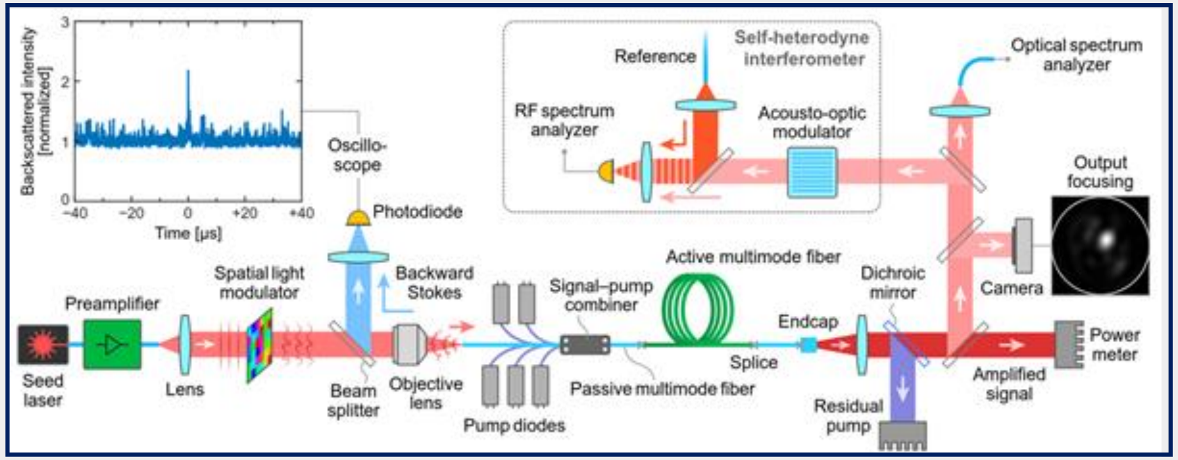
Programmable nonlinear photonics



Yanagimoto et al. *Nature* (2025)



Wu et al. *Nat. Photonics* (2025)



Rothe et al. *Science* (2025).

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Controlling noise of bright fields?

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Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Controlling noise of bright fields?

Typically used for few-photon fields, but here our interest is in bright fields (ultrafast, high-energy laser sources). The question: **are the mean-field and noise (Bogoliubov) transformation simultaneously programmable by means of programming the initial conditions**?

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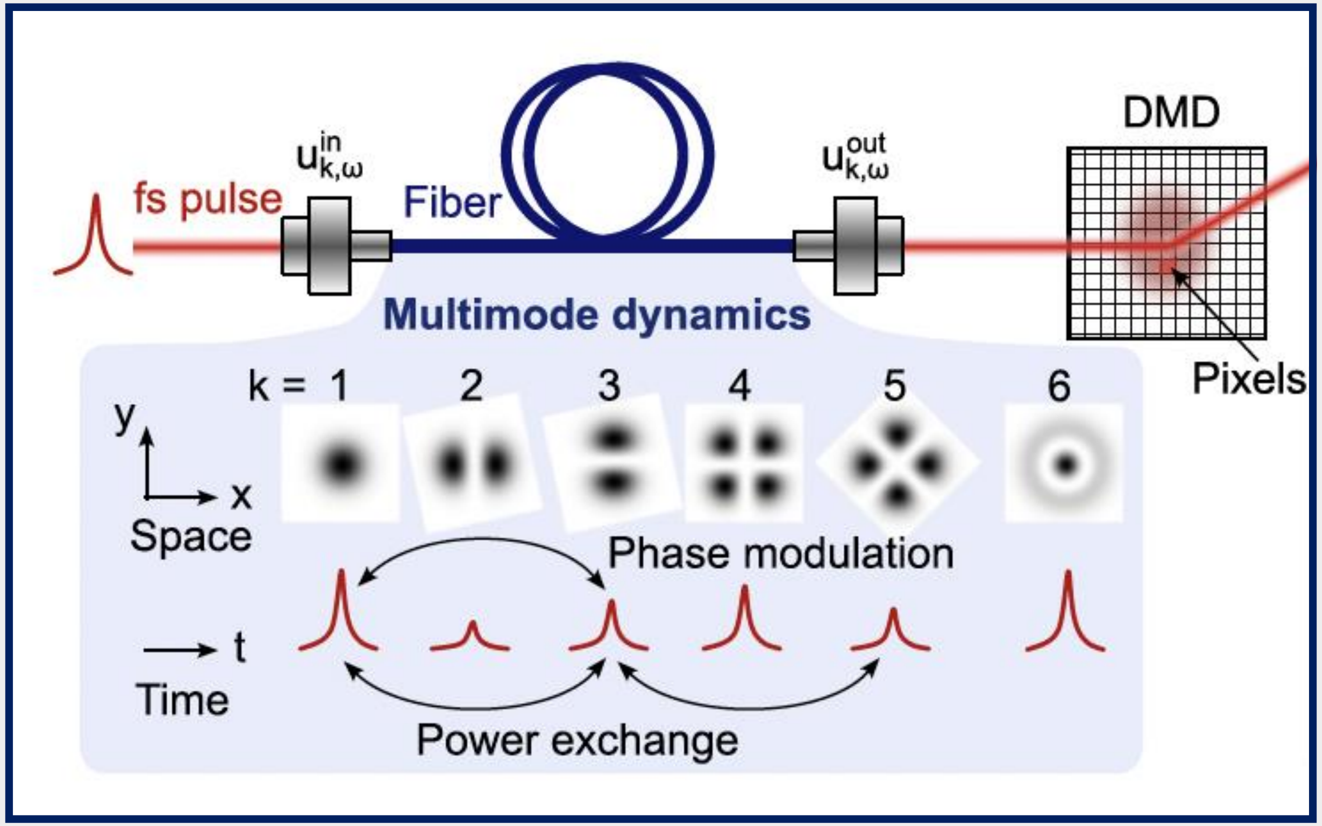
Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Programming intensity and noise



Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

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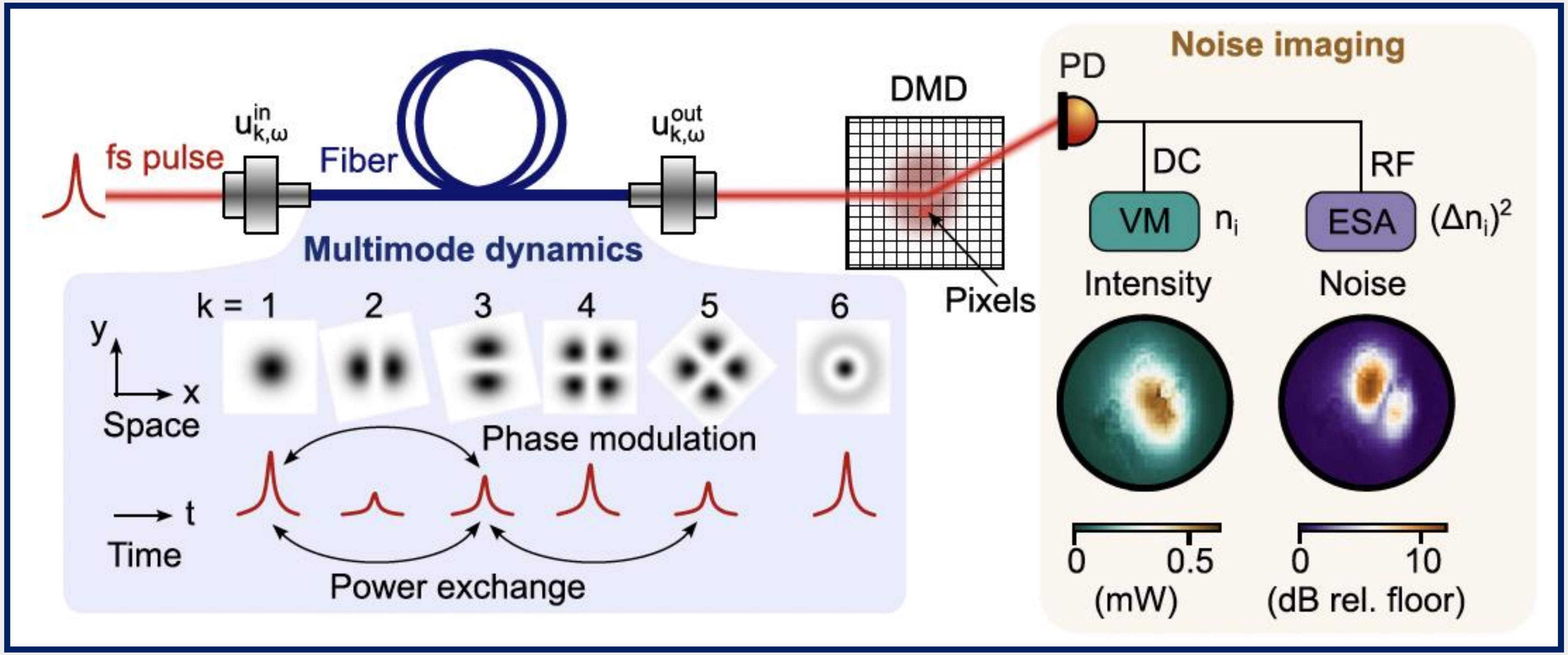

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Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

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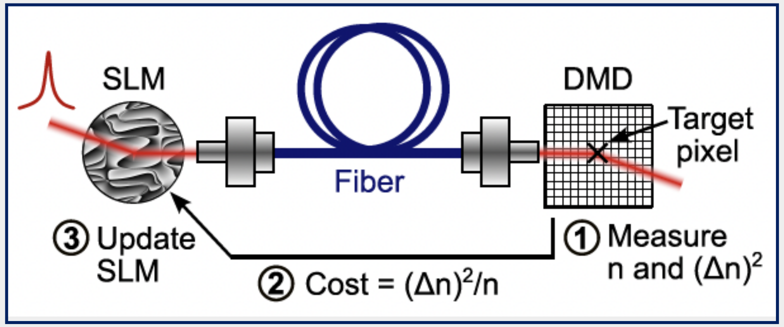

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

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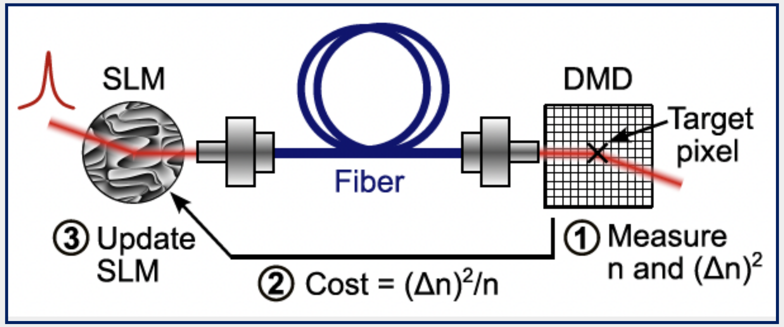
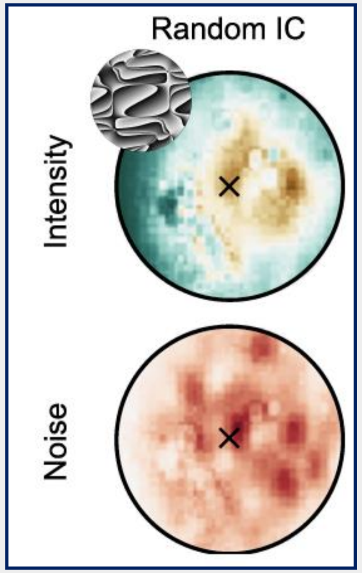
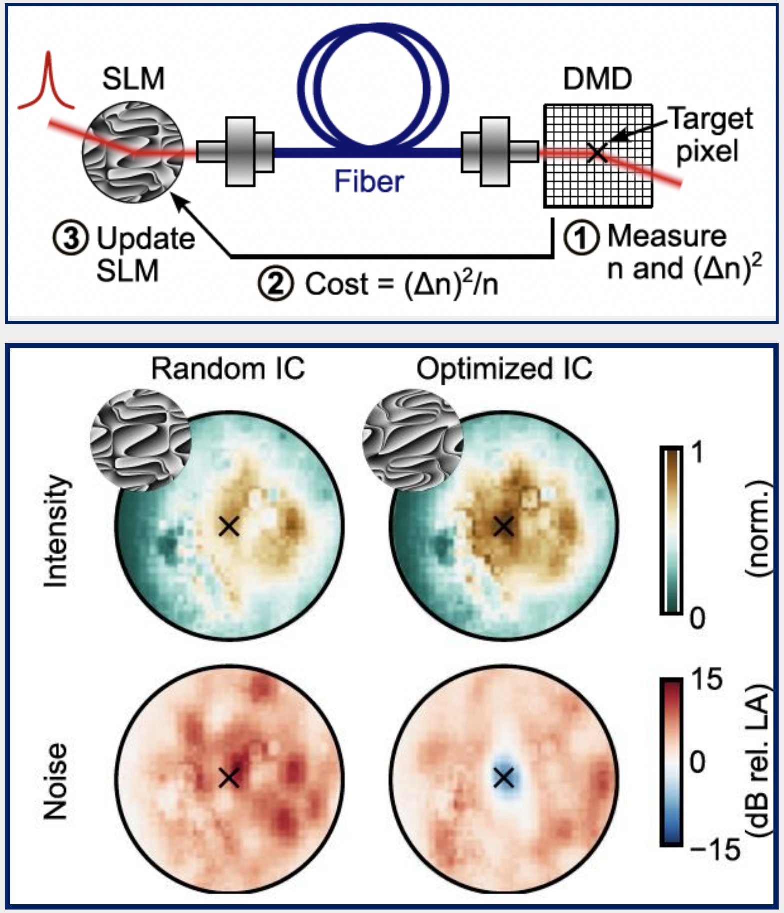




- We simultaneously control the mean intensity and intensity fluctuations by means of the SLM phase profile imprinted on the input.

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Programming intensity and noise



- We simultaneously control the mean intensity and intensity fluctuations by means of the SLM phase profile imprinted on the input.
- Intensity of the beam "center" can stay fixed while noise drops by almost 15 dB relative to linear attenuation, leading to a shot noise limited beam.
  - Simulations based on direct noise optimization (using gradients) show the very same effect with similar magnitude.

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

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Decoupling beam from input noise

$$(\Delta n)^2 = n - \Phi n + \sum_{m}^{M} \left| \frac{\partial n}{\partial u_m^{(0)}} \right|^2 + \delta F_{\text{in}} \left| \sum_{m}^{M} U_m \frac{\partial n}{\partial u_m^{(0)}} \right|^2$$

variance of photon #

bound set by higher-order modes we don't address: sub-shot noise

sensitivity of photon # to initial shot noise in amplitudes

sensitivity of photon # to single-mode *excess noise*

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Laser stabilization perspective

- Stabilization by instantaneous Kerr
  - Broadband in RF domain effectively
  - Can suppress intensity fluctuations *below* shot noise
- Harder to control than active techniques
  - But, autonomous execution of a gradient-free optimization helps: convergence after small # steps here!
- In some sense we're making an mode-selective nonlinear optical limiter
  - But in highly multimode systems, such a limiter is created by the waveform.

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

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  - Can suppress intensity fluctuations *below* shot noise
- Harder to control than active techniques
  - But, autonomous execution of a gradient-free optimization helps: convergence after small # steps here!
- In some sense we're making an mode-selective nonlinear optical limiter
  - But in highly multimode systems, such a limiter is created by the waveform.

Sloan*, Horodynski*, Uddin*, …, and **Rivera**. arXiv: 2509:03482.

Upshots and outlook

- Understanding noise generation and suppression mechanisms in multimode nonlinear optics is a very rich problem
  - Sensitivity analysis gives direct understanding of which noise channels influence a *particular* observable, and enables optimization and inverse design.
  - Intuition from single-mode cases often doesn't carry over. This is an opportunity!
- Highly multimode systems present opportunities for control
  - Co-control of mean-field and Bogoliubov transformation by wavefront shaping.
- Some thoughts for the future:
  - Dimensionality reduction for tractable non-perturbative descriptions of noise
  - Exploring nonlinear induced noise limits of nonlinear amplifiers
  - Creating sub-shot noise resources at extremely high pulse energies (> uJ, mJ…)

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Kavli Institute for Nanoscale Science

If you'd like to revisit anything I shared

Slides from the talk (up to small modifications).

Repository with our adjoint model and examples.

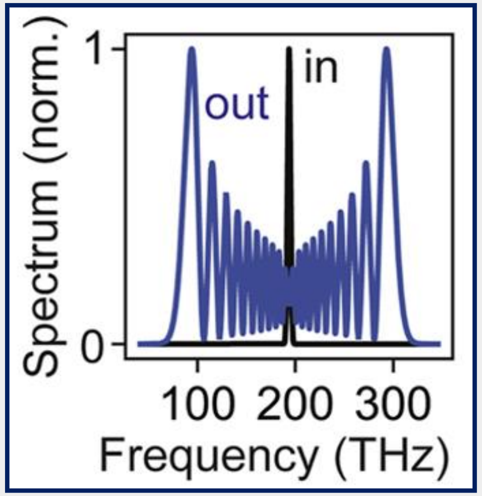
E-mail: nrivera@cornell.edu

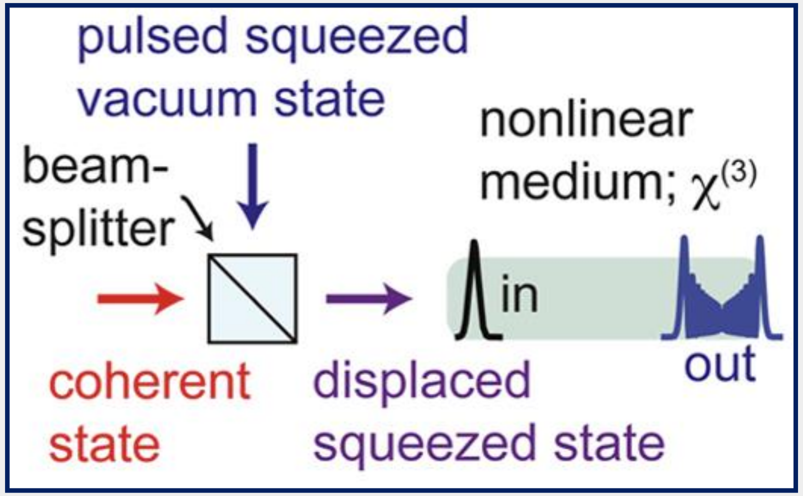
Supplemental slides

Lightning round: Multimode noise behaviors can look very different

Joint number-phase noise suppression

Continuum generation driven by displaced squeezed vacuum: **which noise gets reduced?**

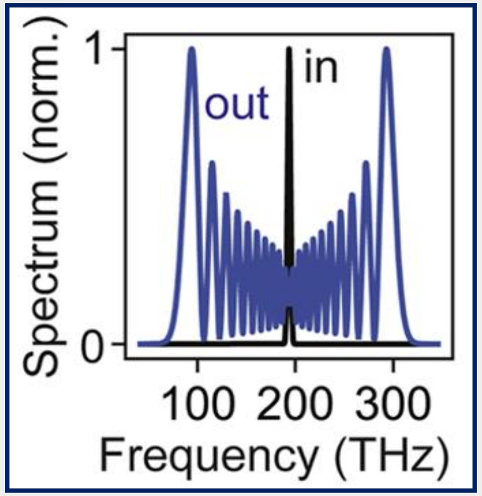


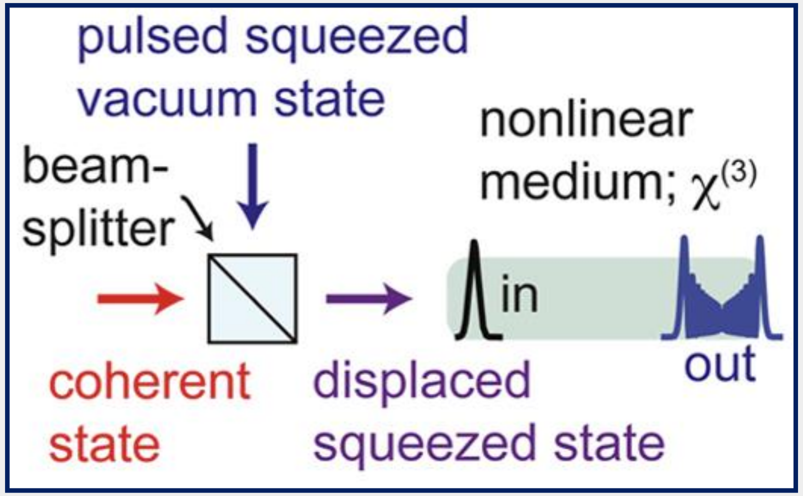


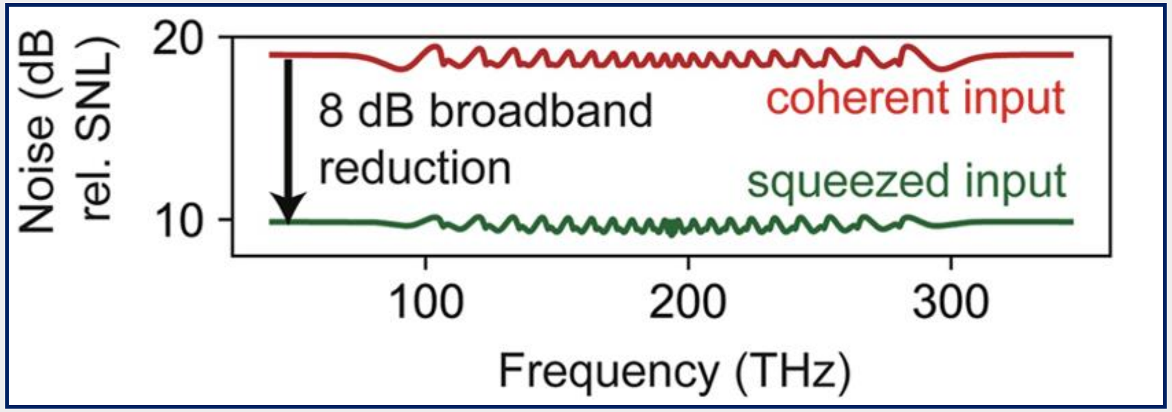
Rivera *et al*. Nanophotonics (2025).

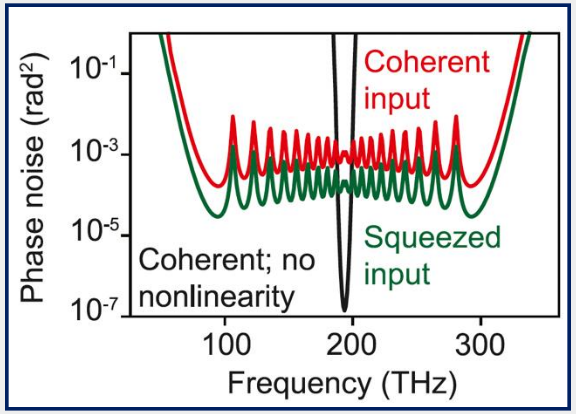
Joint number-phase noise suppression

Continuum generation driven by displaced squeezed vacuum: **which noise gets reduced?**





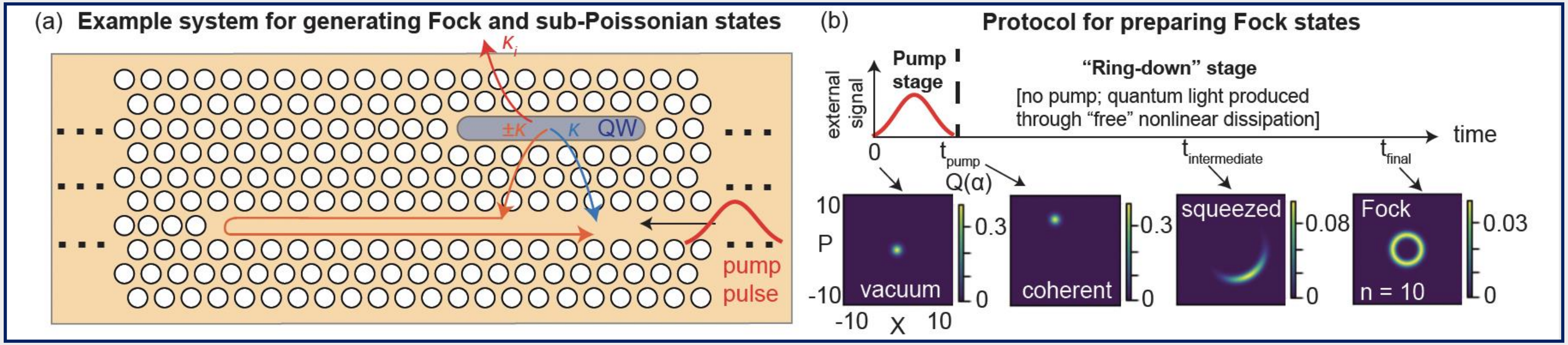


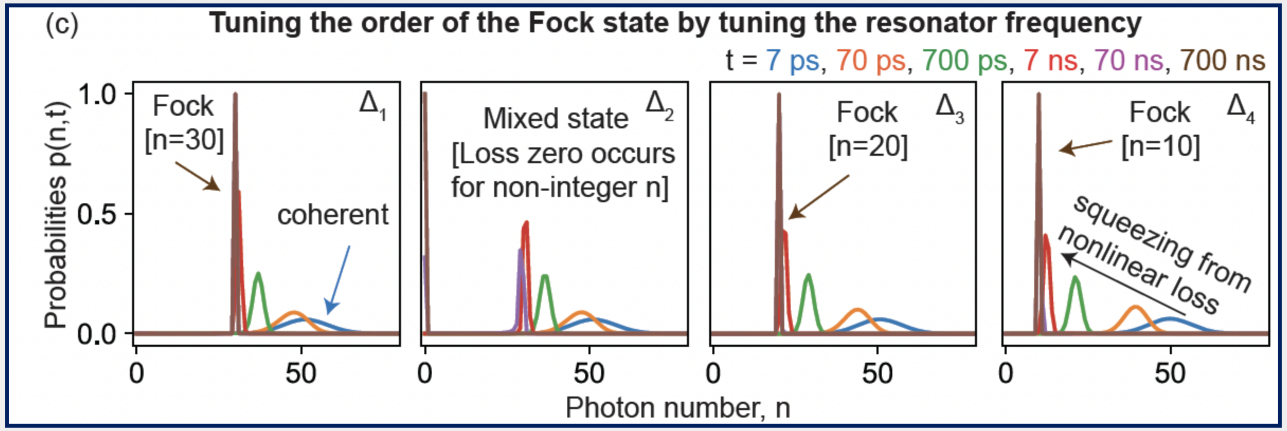


Number and phase couple to same input quadrature; both reduce.

Rivera *et al*. Nanophotonics (2025).

Passive non-Gaussian state stabilization





Mechanism: a photon-number-dependent wave interference leads to a Fock state being the fixed point of the dynamics.

Rivera *et al*. PNAS (2023).

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Seeding light sources with squeezed pulses to improve performance

Rivera *et al.* Nanophotonics (2025).

Quantum noise amplifying nonlinearity

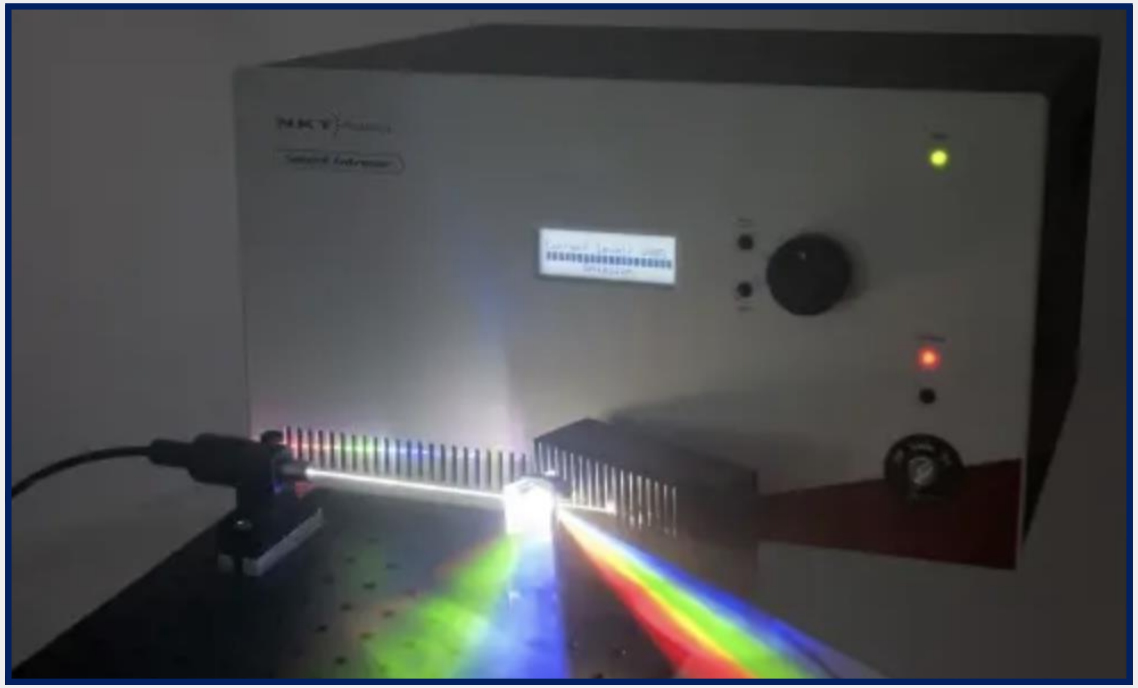
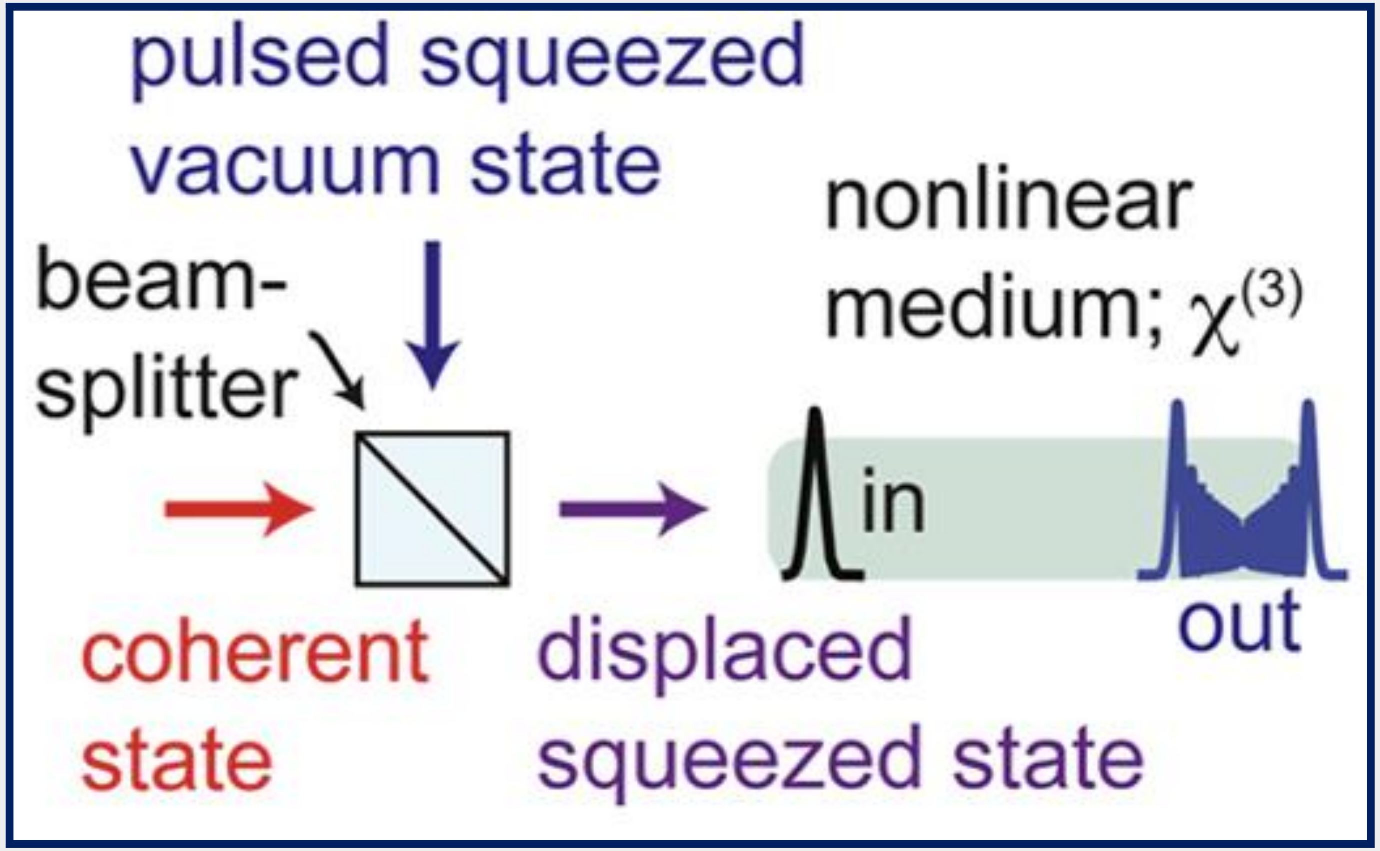


Image credit: NKT Photonics.

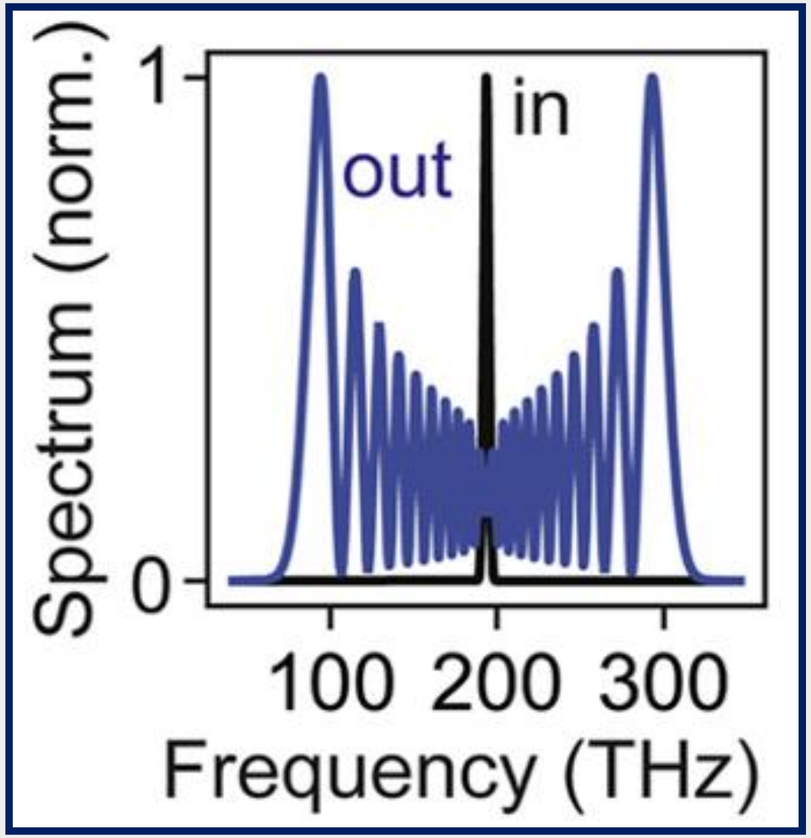
Many nonlinear optical phenomena strongly amplify small changes in initial conditions (noise) such that even when the inputs have small quantum noise, the output noise can become comparable to the mean field.

Seeding sources with squeezed pulses



N. Rivera *et al. Nanophotonics (2025).*

Example: self-phase modulation in fiber



Classical self-phase modulation:

$$\partial_z \alpha(z,t) = i\gamma \alpha^*(z,t)\alpha^2(z,t)$$

Time-domain solution:

$$\alpha(z,t) = e^{i\gamma z|\alpha(0,t)|^2}\alpha(0,t)$$

Classical spectrum:

$$n(L,\omega) = |\alpha(L,\omega)|^2$$

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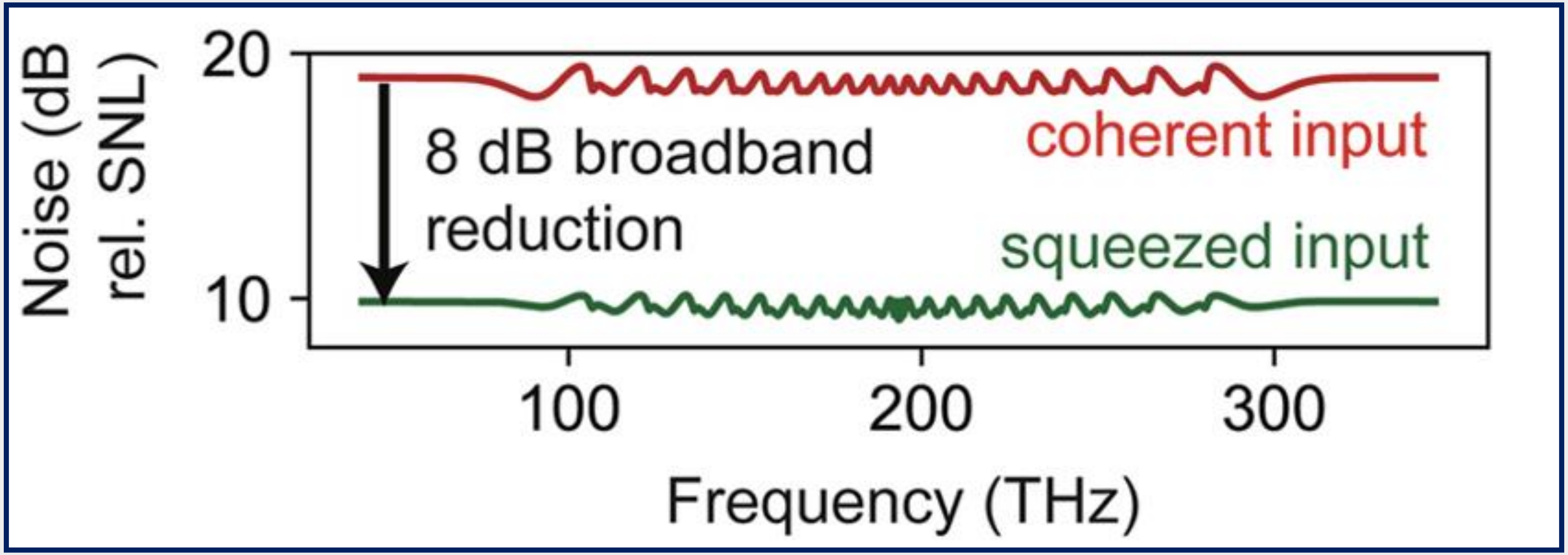
Suppression of spectral intensity noise



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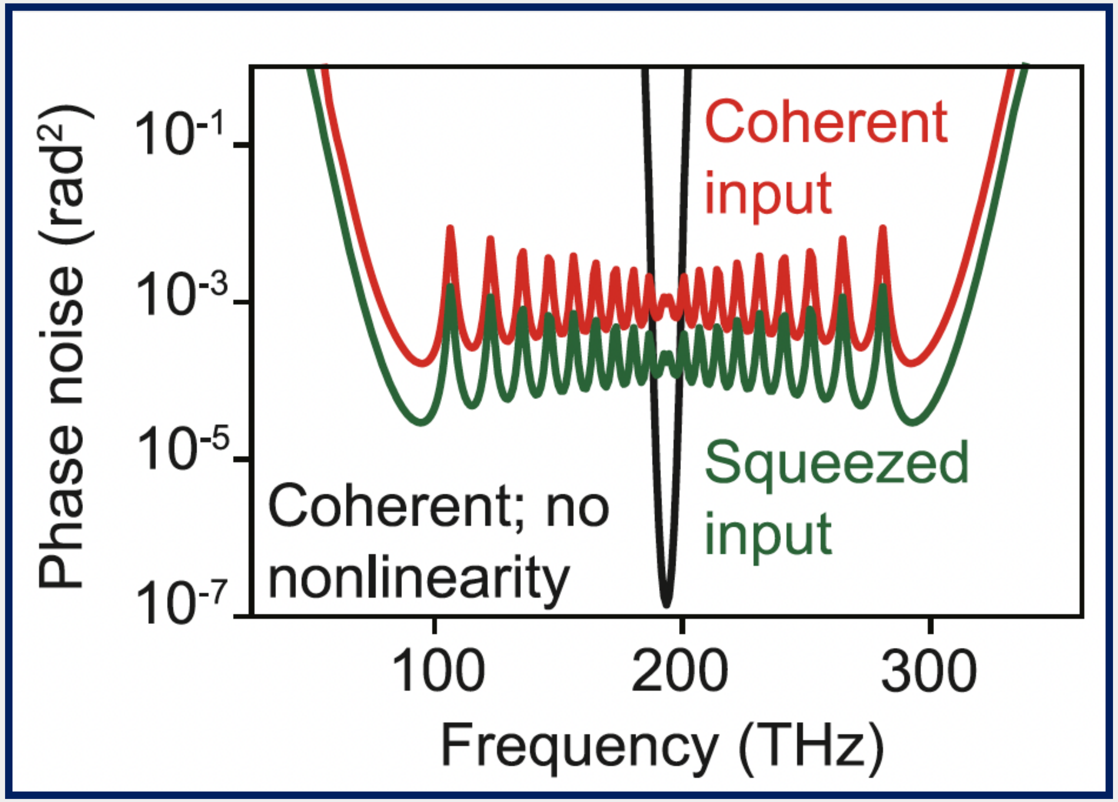
Suppression of spectral phase noise



Both noises are simultaneously reducible in a broadband way because they both emerge from amplification of input noise in the pulse in the same ("amplitude") quadrature.

N. Rivera *et al. Nanophotonics (2025).*

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