AEP 4400/5400 Spring 2026: Nonlinear and Quantum Optics – Intro

Nick Rivera

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Today: logistics & overview

Administrative info. (all on syllabus)

Instructor: Nick Rivera; nrivera@cornell.edu; Clark 206
Grader: Marcel Latasa; mrl259@cornell.edu

Course time: 9:05-9:55am MWF, Rockefeller 122
Office hours: 2-3pm Wednesdays, Clark 206

Course organization: Lecture-based, six problem sets (worth 25%), two midterms (15% each), one final exam (30%). Occasional mini-quizzes in class (worth 5%). Attendance and engagement is important (worth 10%)!

For all info on key course dates and policies regarding homework, collaboration, and AI, please consult syllabus (on Canvas).

Course info. (all on syllabus)

Topics: ~60% classical nonlinear optics including all canonical topics, ~40% quantum optics including electromagnetic field quantization, the quantum theory of nonlinear optics, and applications to quantum information science.

Texts:

- Nonlinear Optics, Fourth Edition, Robert Boyd (required)
- Course notes (distributed prior to class; no substitute for attendance!)
- Other optional texts suggested on syllabus

Prereqs: Intermediate electromagnetism at the level of AEP 3560, intermediate quantum mechanics at the level of AEP 3620, and mathematical physics at the level of AEP 4200.

What is this course about?

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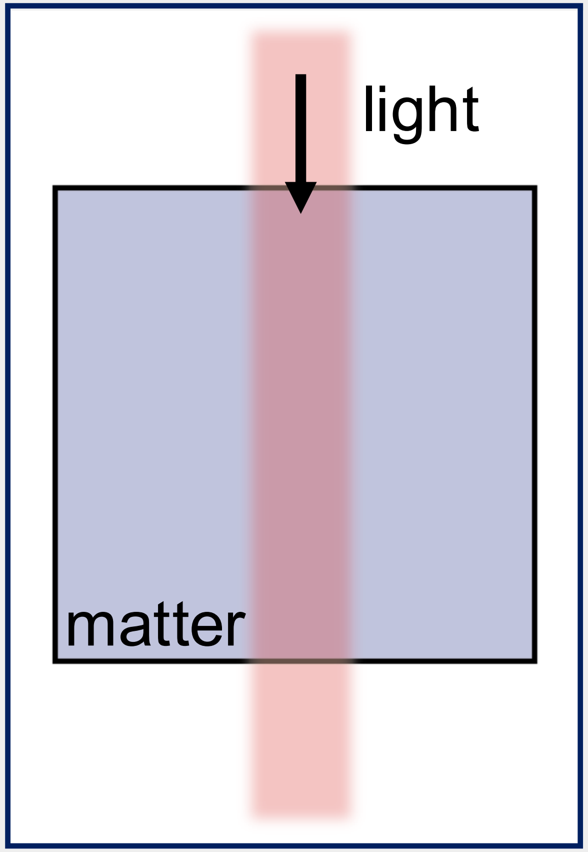
The interaction between light and matter.

Part I: Classical nonlinear optics

Linear response of materials to light



Light-induced polarization:

$$P_i = \epsilon_0 \chi_{ij}^{(1)} E_j$$

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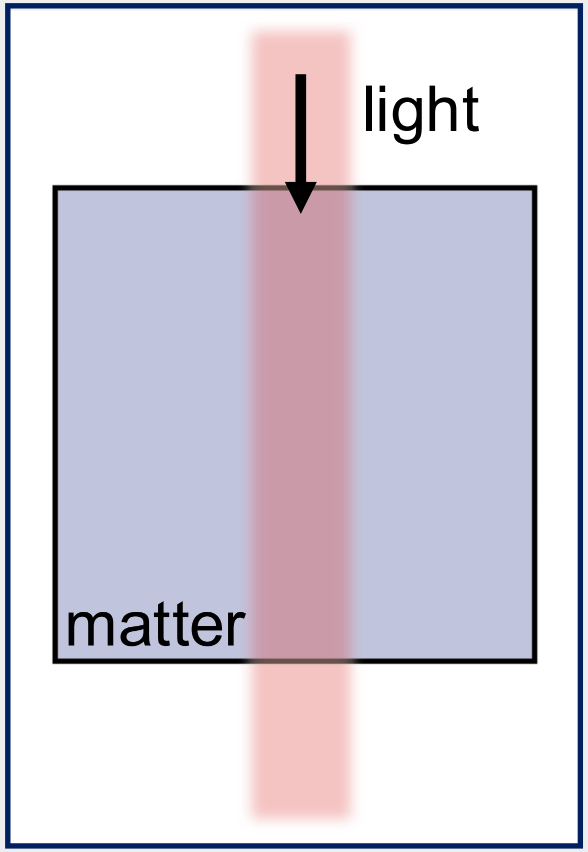
Linear response of materials to light



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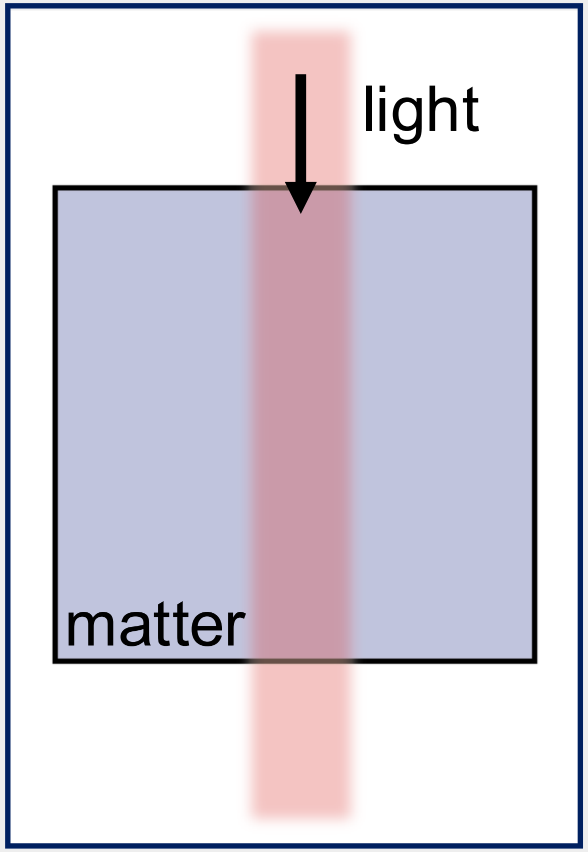
Canonical example: Lorentz oscillator

$$\ddot{\mathbf{x}}(t) + \gamma \dot{\mathbf{x}}(t) + \omega_0^2 \mathbf{x}(t) = q\mathbf{E}(\mathbf{r}, t)/m$$

Linear response of materials to light



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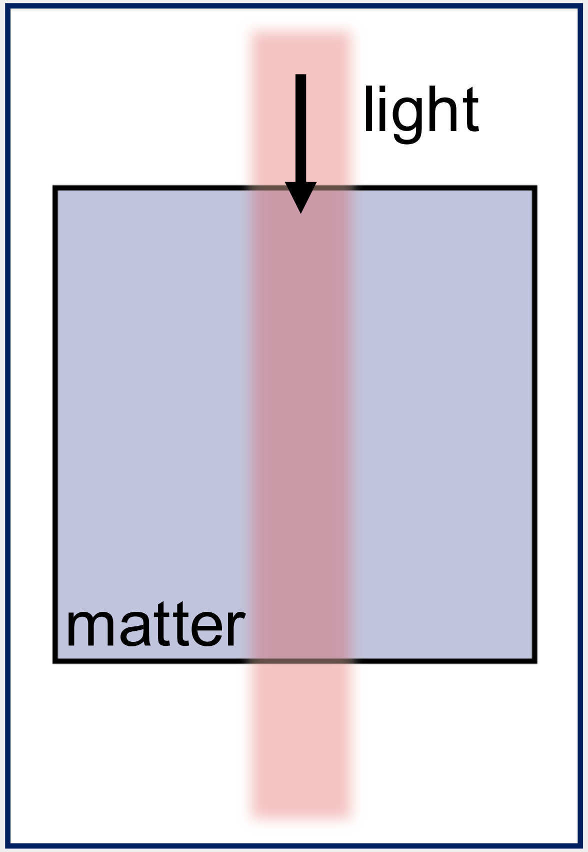
Position responds linearly to field:

$$\mathbf{x}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \frac{q\mathbf{E}(\mathbf{r}, \omega)/m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Linear continuum electromagnetism

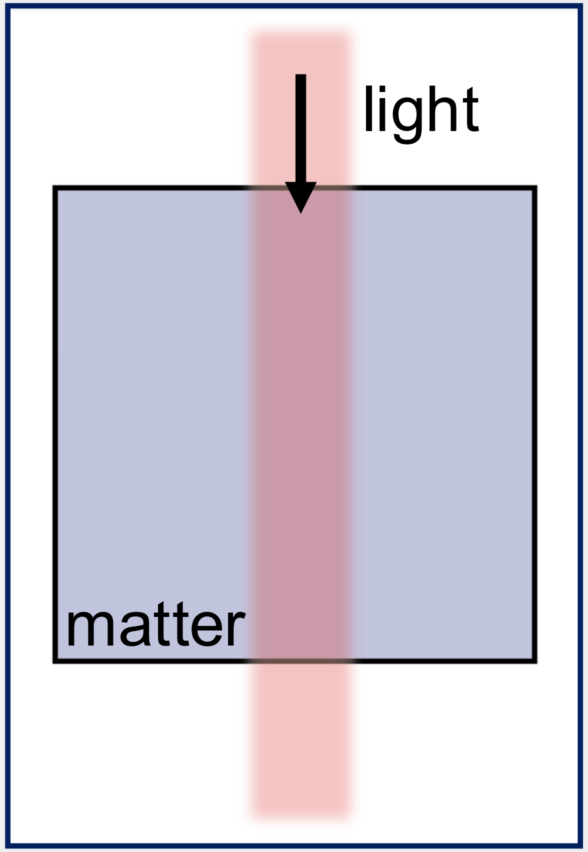


Maxwell's equations in medium:

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho \qquad \nabla \times \mathbf{E}(\mathbf{r},t) = -\mu_0 \mathbf{H}(\mathbf{r},t)$$

$$\nabla \cdot \mathbf{H}(\mathbf{r},t) = 0 \qquad \nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) + \partial_t \mathbf{D}(\mathbf{r},t)$$

Linear continuum electromagnetism



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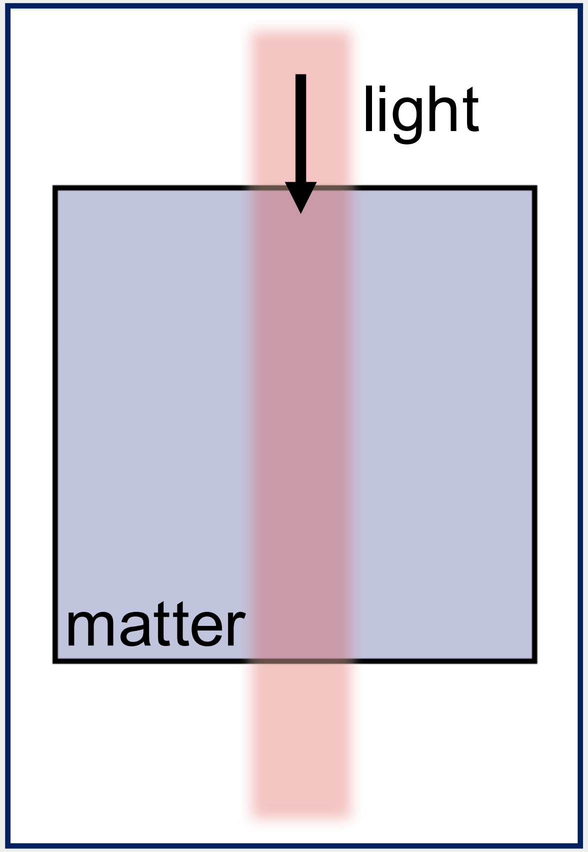
Constitutive relation for nonmagnetic medium:

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega)$$

$$\epsilon(\omega) = \epsilon_0(1 + \chi^{(1)}(\omega))$$

Linear continuum electromagnetism



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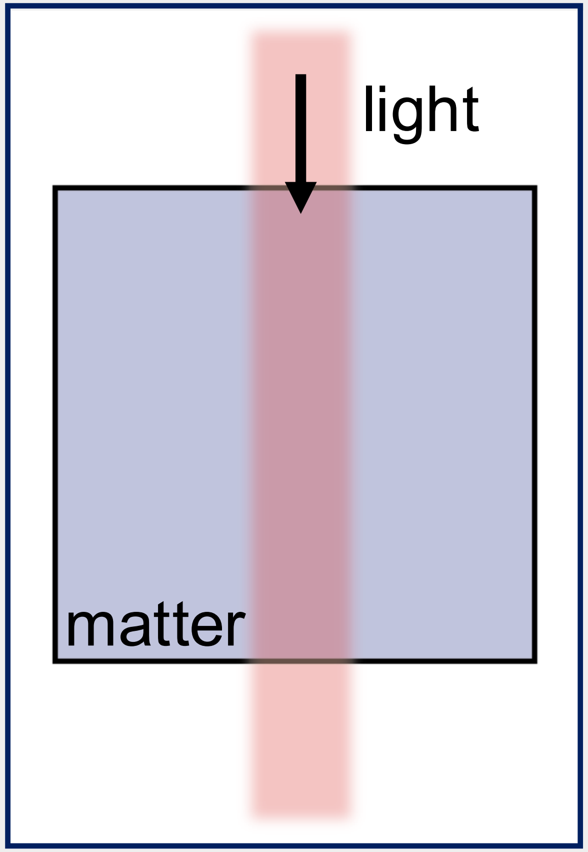
Maxwell wave equation with a source:

$$\left[\nabla \times \nabla \times -\epsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\right]\mathbf{E}(\mathbf{r},\omega) = -\frac{\omega^2}{\epsilon_0 c^2}\mathbf{P}(\mathbf{r},\omega)$$

These equations are linear!

Nonlinear response of materials to light



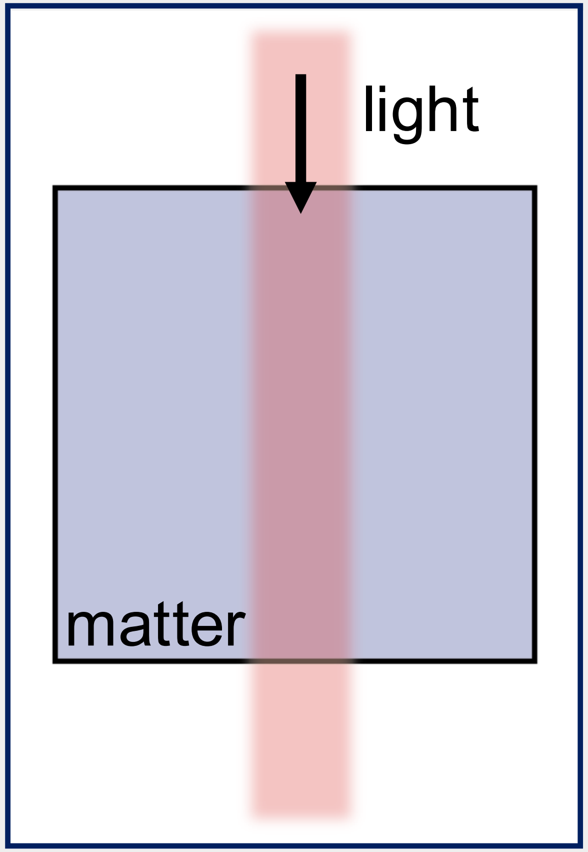
Light-induced polarization:

$$P_i = \epsilon_0 \left(\underbrace{\chi^{(1)}_{ij} E_j}_{\text{Weak fields}} + \underbrace{\chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \dots}_{\text{Stronger driving fields (e.g., using focused lasers)}} \right)$$

Weak fields

Stronger driving fields (e.g., using focused lasers)

Nonlinear response of materials to light



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Weak fields

Stronger driving fields (e.g., using focused lasers)

No more superposition! Leads to a variety of very important physical consequences.

Manifestations of nonlinear optics

Harmonic generation and green lasers

Second-order polarization:

$$P^{(2)} = \epsilon_0 \chi^{(2)} E^2$$

Harmonic generation and green lasers

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$$E = E_0 e^{-i\omega t} + \mathrm{c.c}$$

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Induced polarization:

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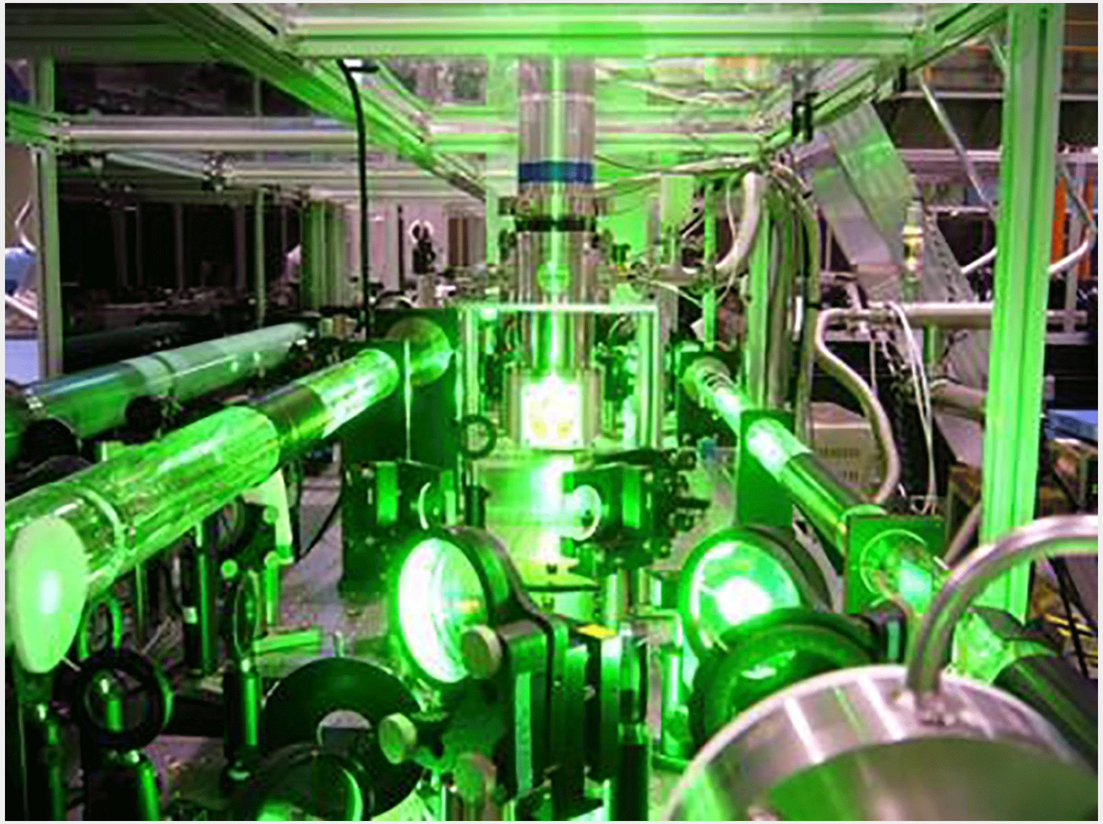
Harmonic generation and green lasers



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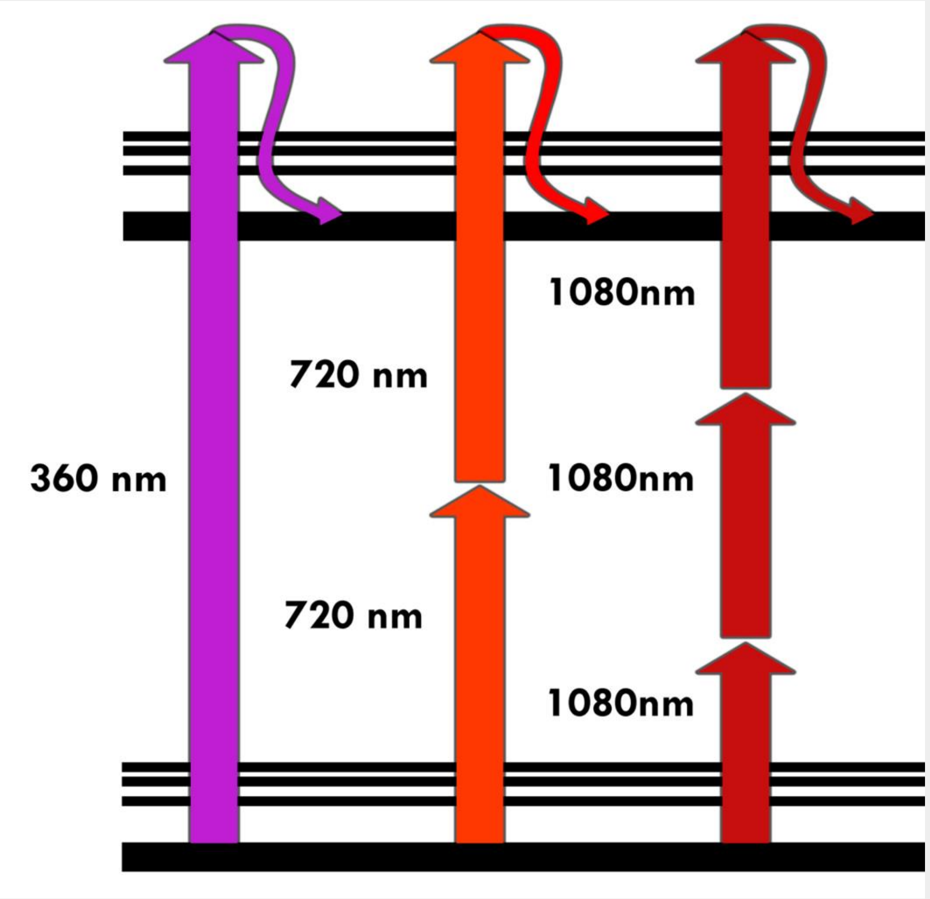
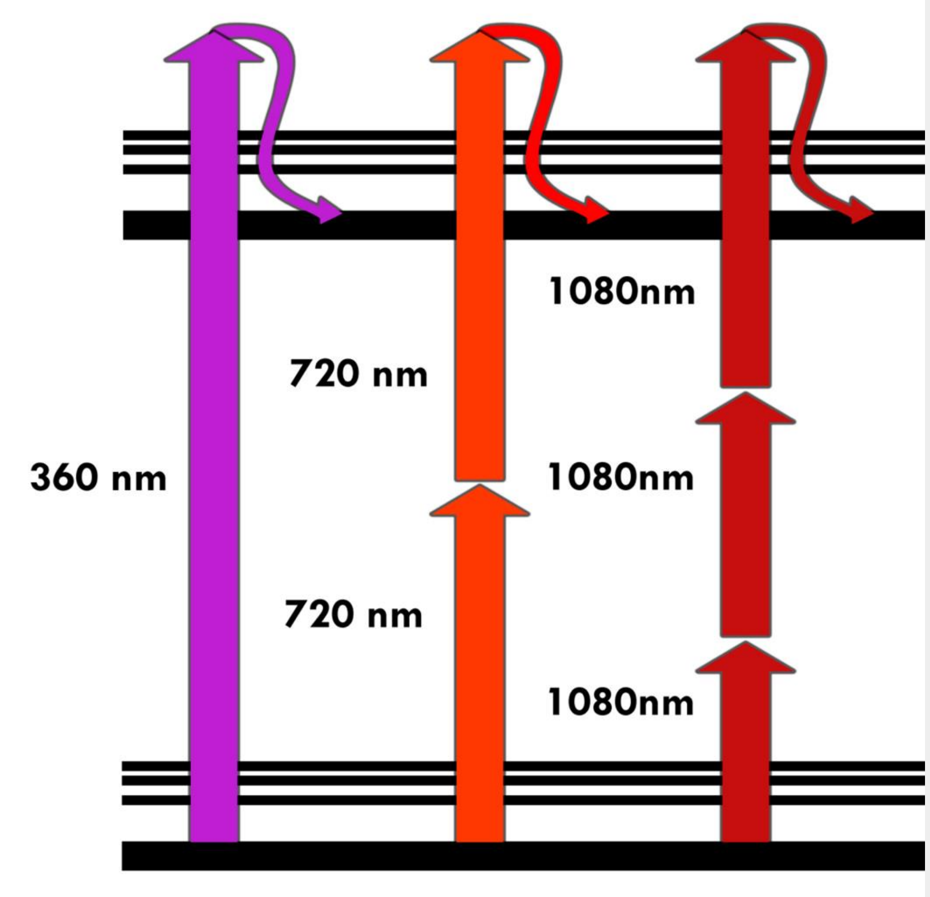
This is how all green lasers are made!

Multiphoton absorption



A system can absorb more than one photon provided that energy conservation is satisfied!

Multiphoton absorption



$$R_{mg}^{(1)} = \left| \frac{\mu_{mg} E}{\hbar} \right|^2 2\pi \rho_f (\omega_{mg} - \omega),$$

$$R_{ng}^{(2)} = \left| \sum_m \frac{\mu_{nm} \mu_{mg} E^2}{\hbar^2 (\omega_{mg} - \omega)} \right|^2 2\pi \rho_f (\omega_{ng} - 2\omega),$$

$$R_{og}^{(3)} = \left| \sum_{mn} \frac{\mu_{on} \mu_{nm} \mu_{mg} E^3}{\hbar^3 (\omega_{ng} - 2\omega)(\omega_{mg} - \omega)} \right|^2 2\pi \rho_f (\omega_{og} - 3\omega)$$

A system can absorb more than one photon provided that energy conservation is satisfied!

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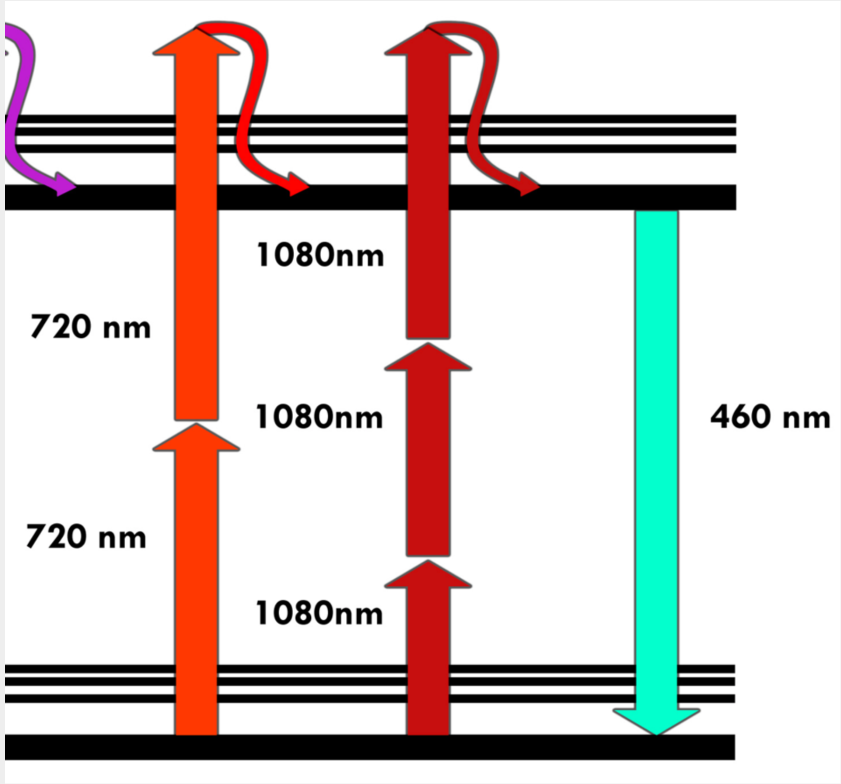
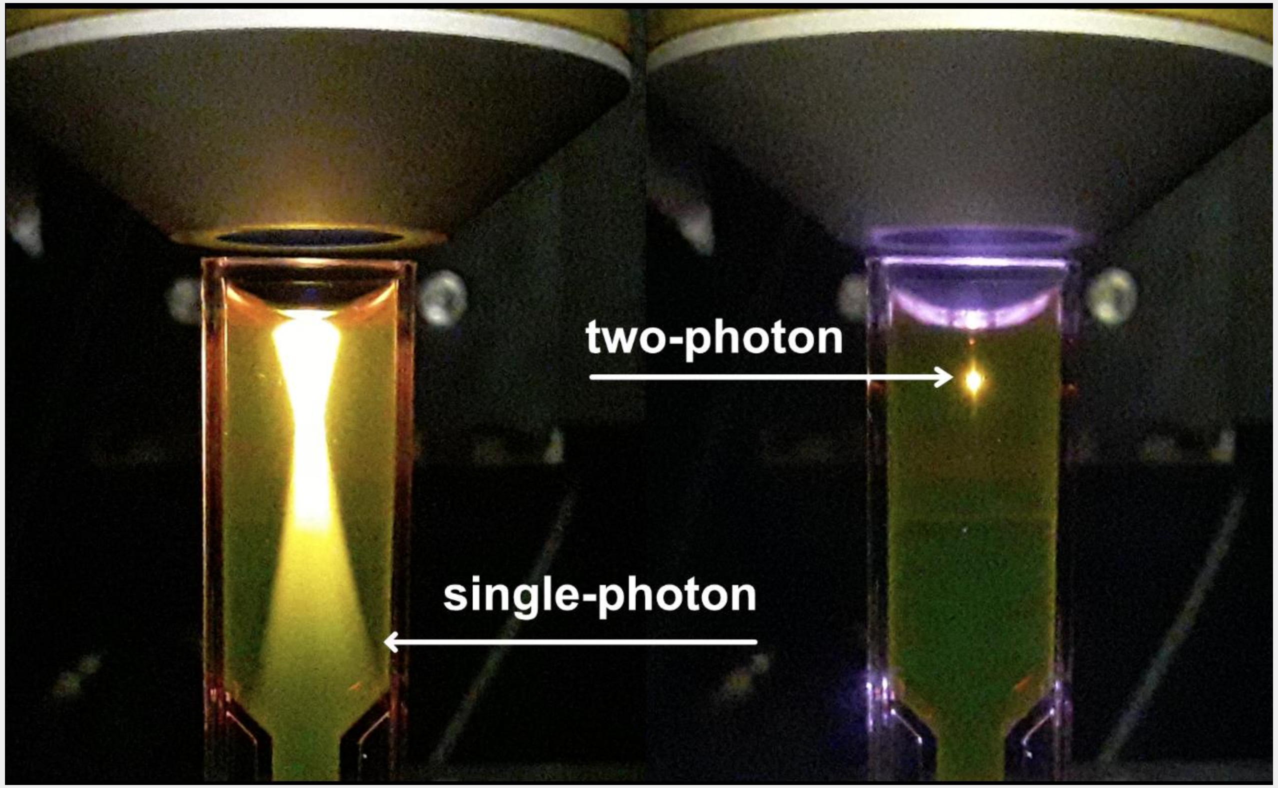
Idea discovered by Goeppert-Mayer in ~1930.

Multiphoton microscopy





This concept is of *major* consequence for deep tissue imaging in biological systems!

Intensity-dependent refractive index

Third-order polarization:

$$P^{(3)} = \epsilon_0 \chi^{(3)} E^3$$

Apply continuous-wave (CW) field:

$$E = E_0 e^{-i\omega t} + E_0^* e^{i\omega t}$$

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Apply continuous-wave (CW) field:

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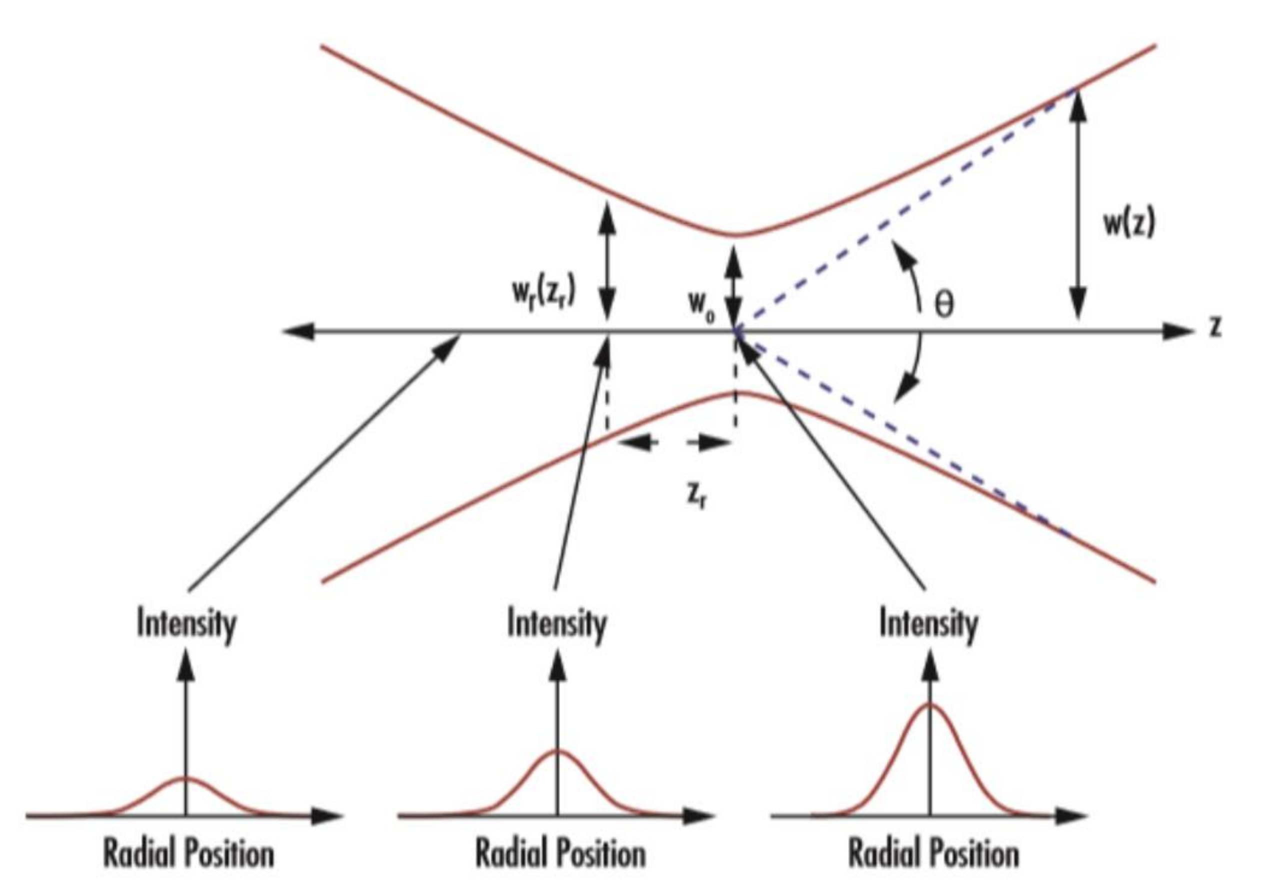
Induced polarization:

$$P(\omega) \sim 3|E_0|^2 E_0 e^{-i\omega t}$$

Acts as an intensity-dependent index of refraction!

Self-focusing of light



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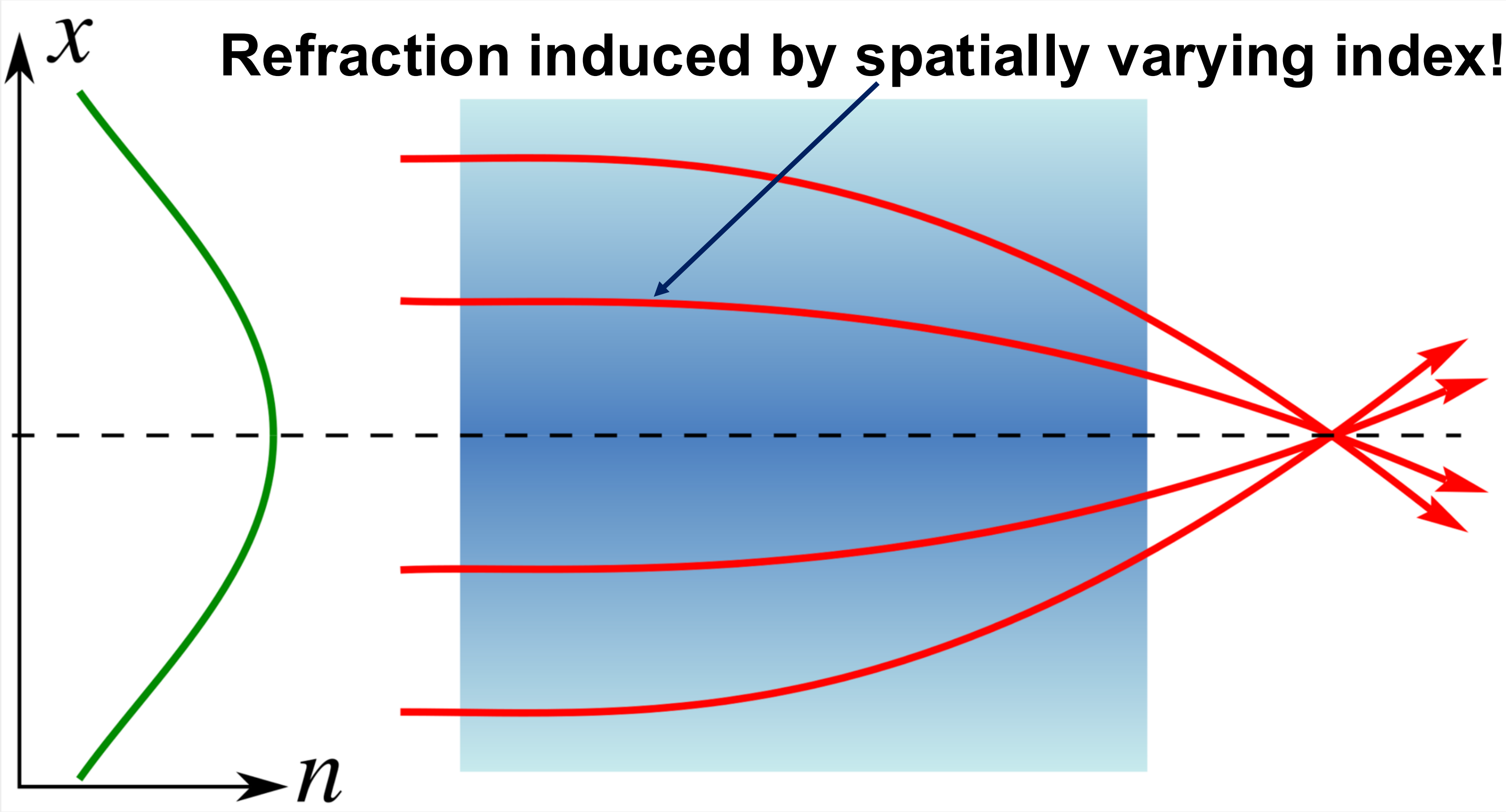

The problem of self-focusing motivated work leading to the 2018 Nobel Prize in Physics.

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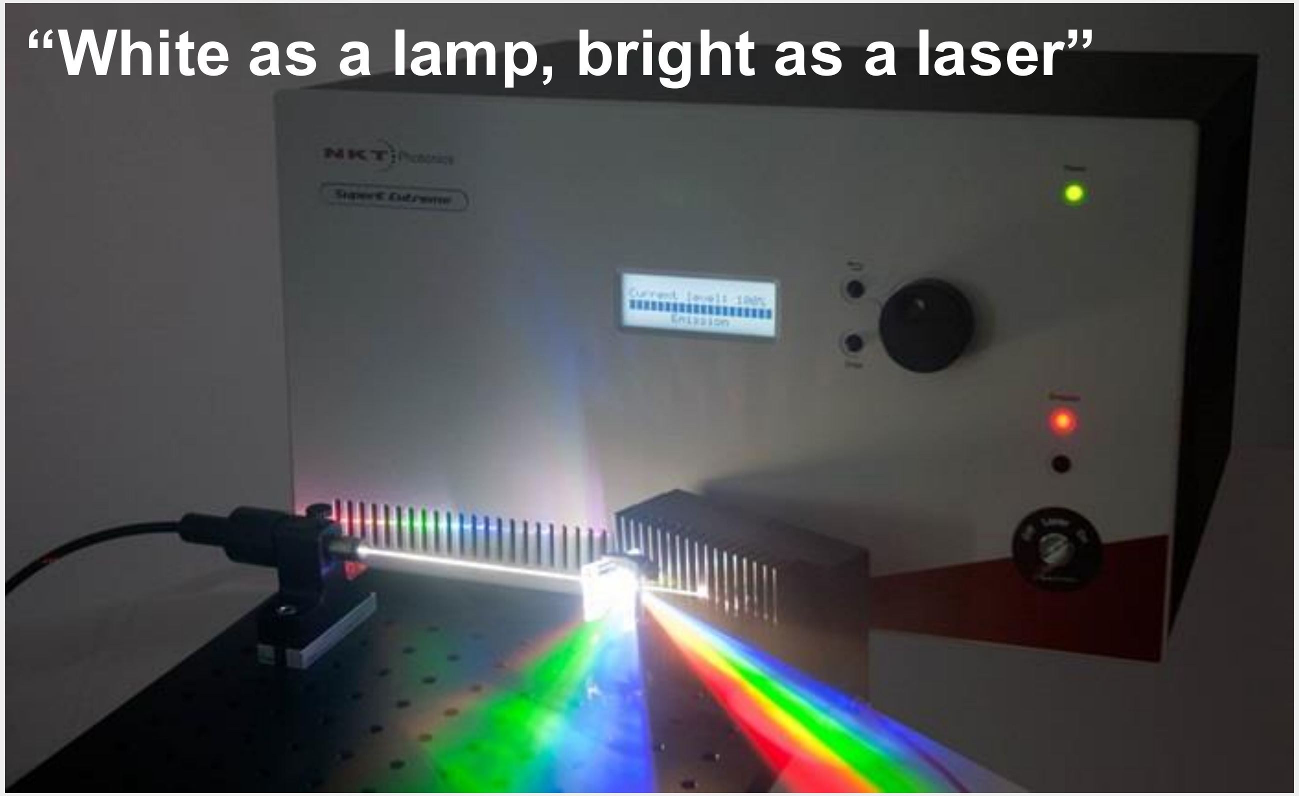
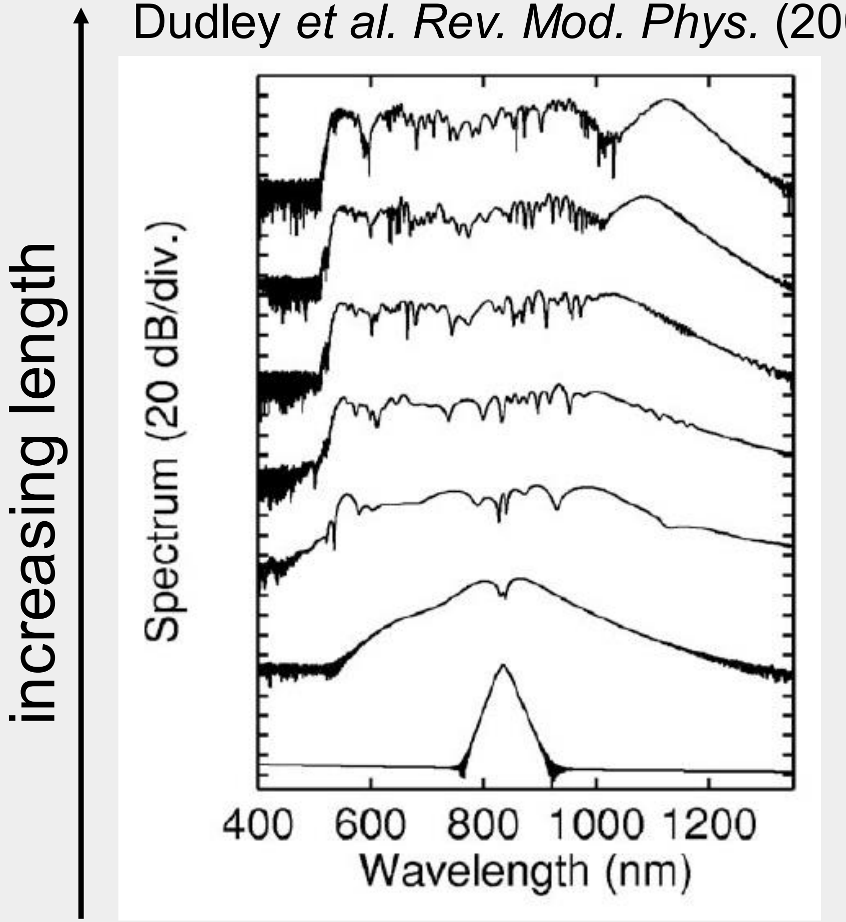
Supercontinuum generation

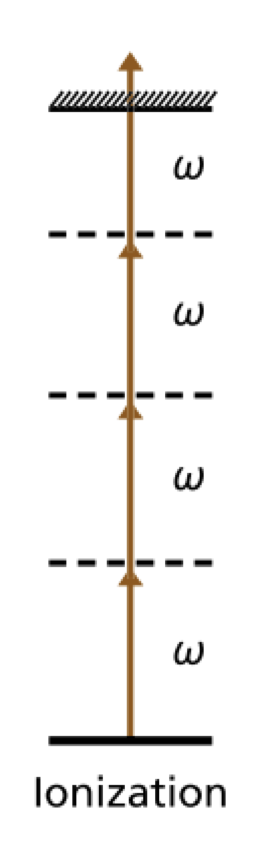


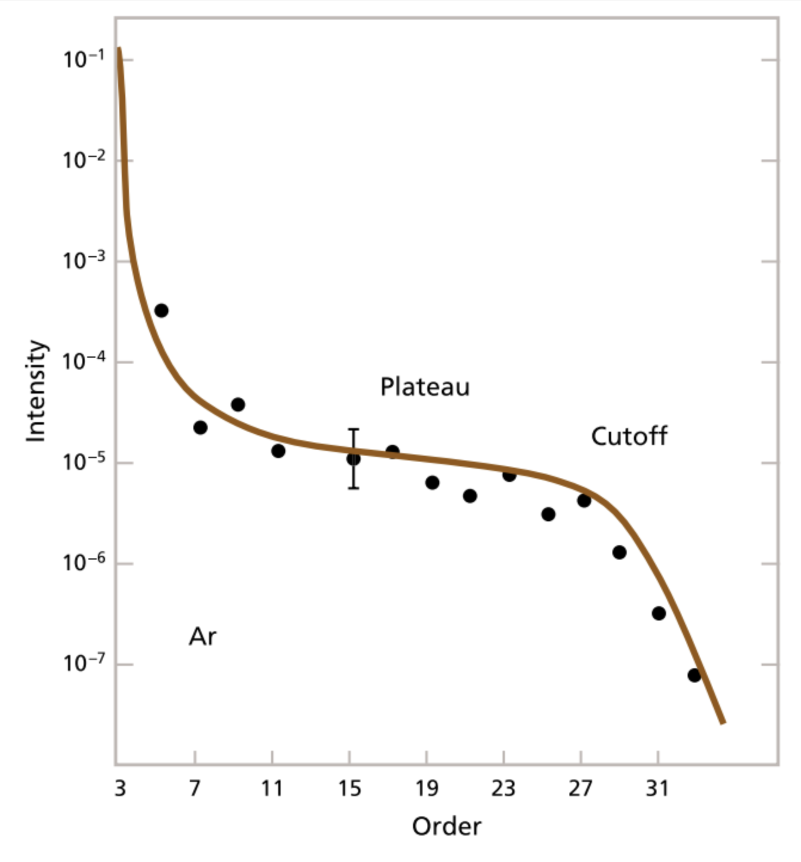
Dudley *et al. Rev. Mod. Phys.* (2006).



This system is of great importance for spectroscopy due to broad wavelength tunability.

High-harmonic generation

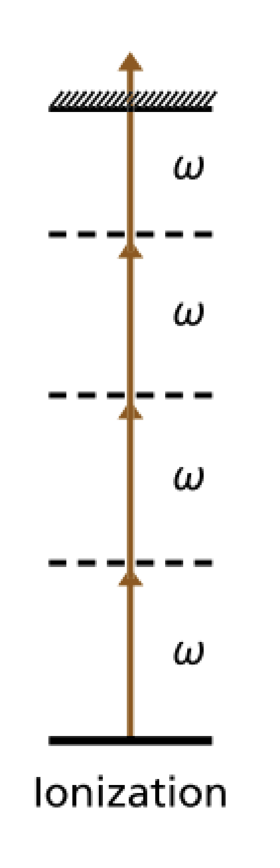


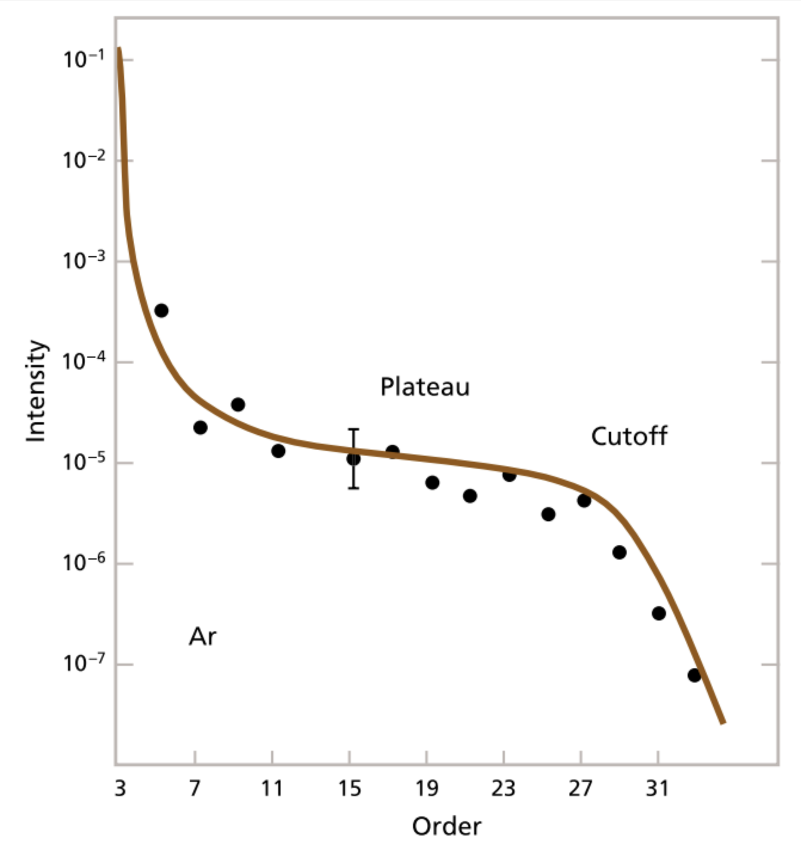


At very high intensities, series expansion for polarization breaks down, leading to so-called *non-perturbative effects*.

L'Huillier 2023 Nobel Lecture.

High-harmonic generation





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This effect can create pulses of *attosecond* duration (10^{-18} s); see 2023 Nobel Prize in Physics.

L'Huillier 2023 Nobel Lecture.

Part II: Quantum optics

Quantization of electromagnetic field

Hamiltonian of the electromagnetic field:

$$H = \frac{\epsilon_0}{2} \int d^3r \; \mathbf{E}^2 + c^2\mathbf{B}^2 = \sum_{k,\lambda} \hbar\omega_{\mathbf{k},\lambda} \left(a^\dagger_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda} + \frac{1}{2} \right)$$

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Each plane-wave mode of light can be thought of as a quantum harmonic oscillator!

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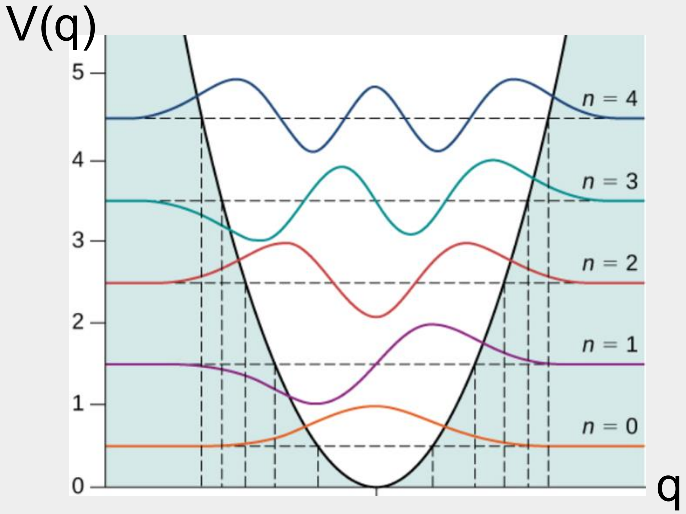

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$

Energy eigenstates:
“n-photon” states

Lasers as coherent states

Idealized representation of single mode laser light, the Glauber coherent state:

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

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with standard variances:

$$\left(\Delta(a + a^\dagger)\right)^2 = \left(\Delta(ia^\dagger - ia)\right)^2 = 1$$

$$(\Delta n)^2 = \langle n \rangle = |\alpha|^2$$

Lasers as coherent states

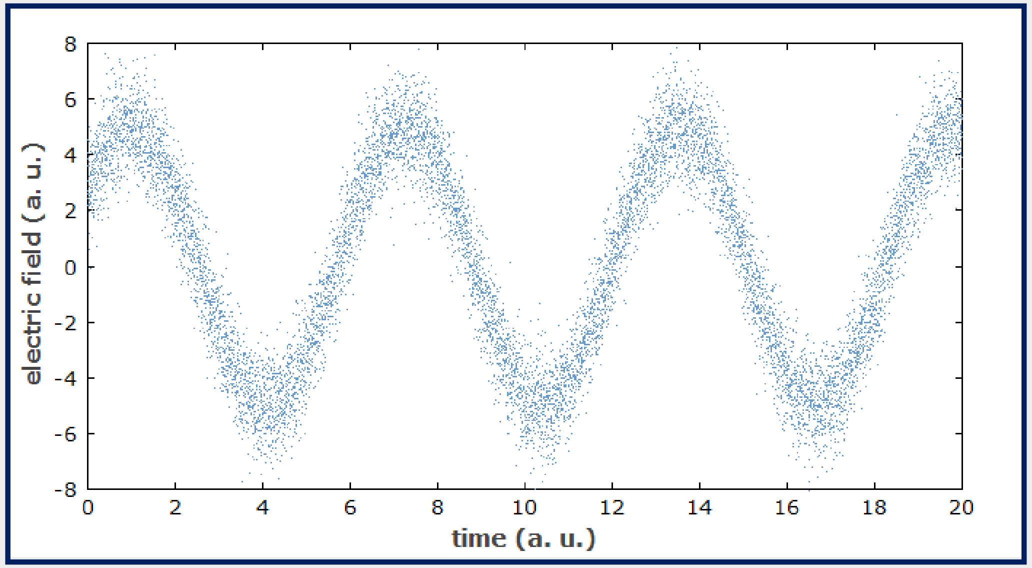

Image credit: RP Photonics

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This understanding is deeply tied to the 2005 Nobel Prize in Physics.

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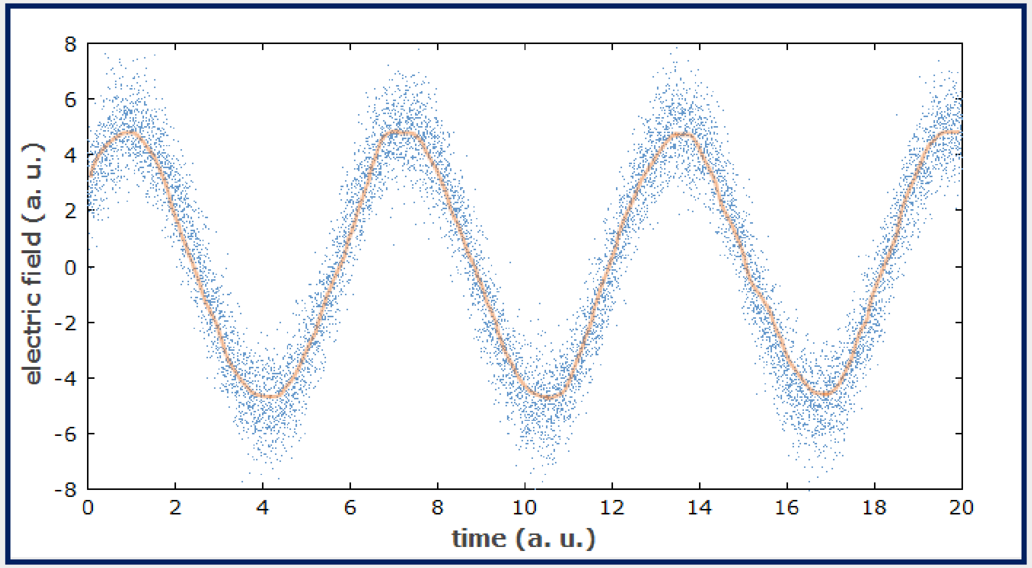

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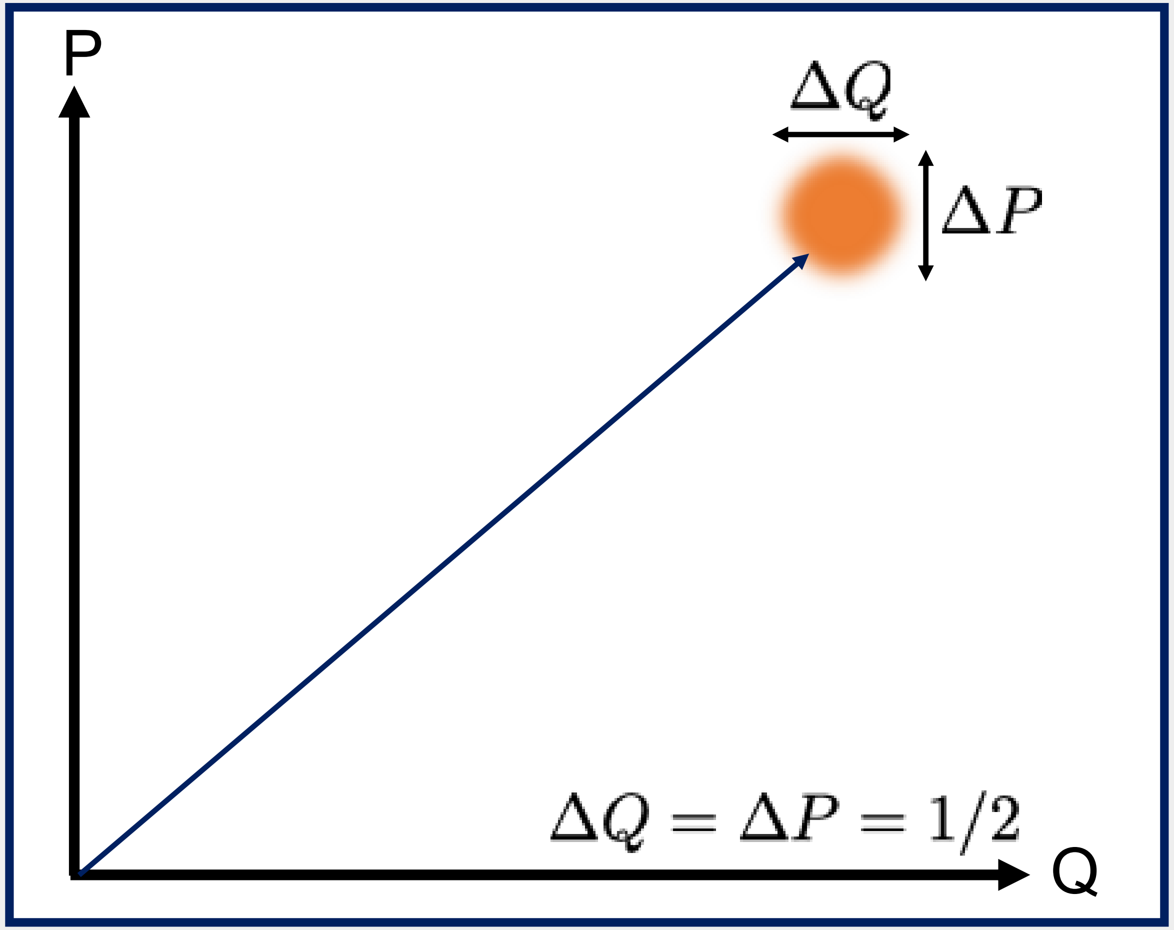
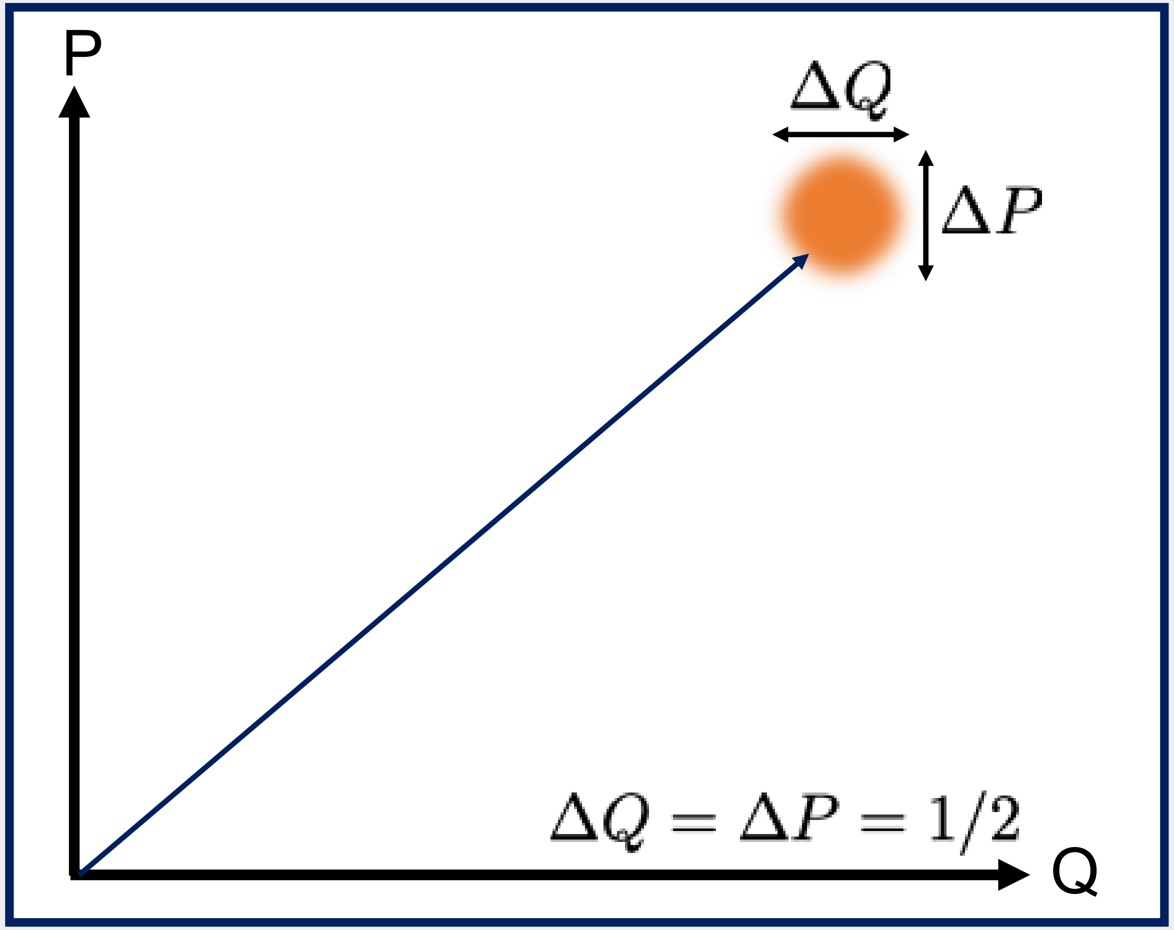
Shot noise in phase space



Shot noise in phase space



Energy (and thus photon number) given by: $n = r^2 = Q^2 + P^2$.

From the diagram:

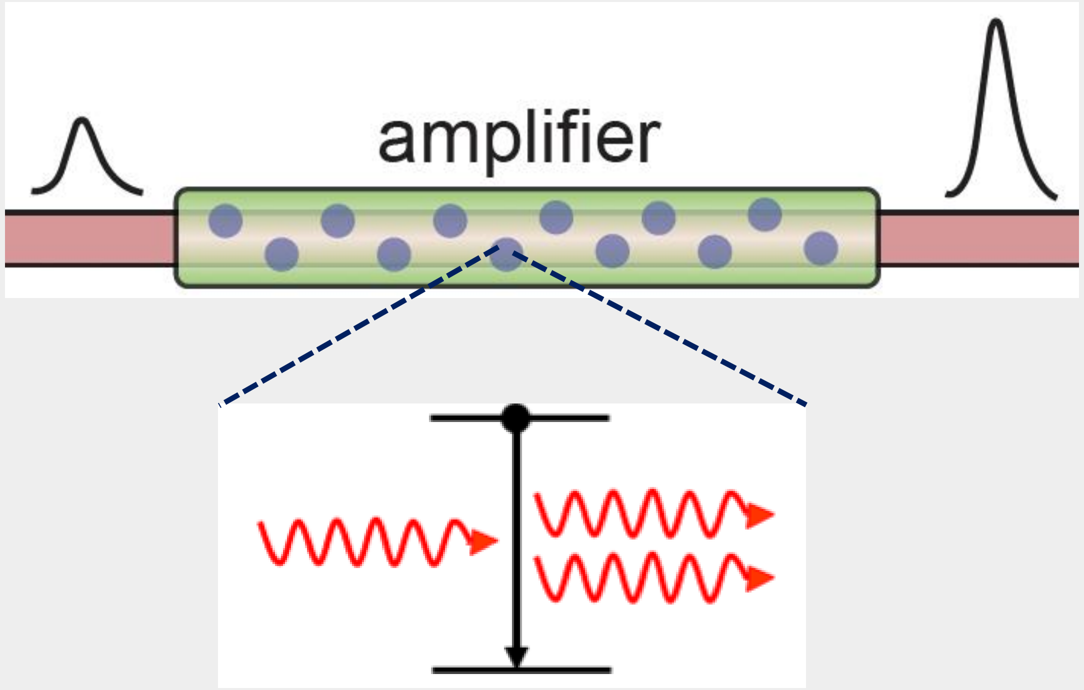
$$\Delta n = \Delta r^2 = 2r\Delta r = \sqrt{n}$$

These intensity fluctuations are called **shot noise,** and result from *superposition of quantum noise with a classical deterministic field*.

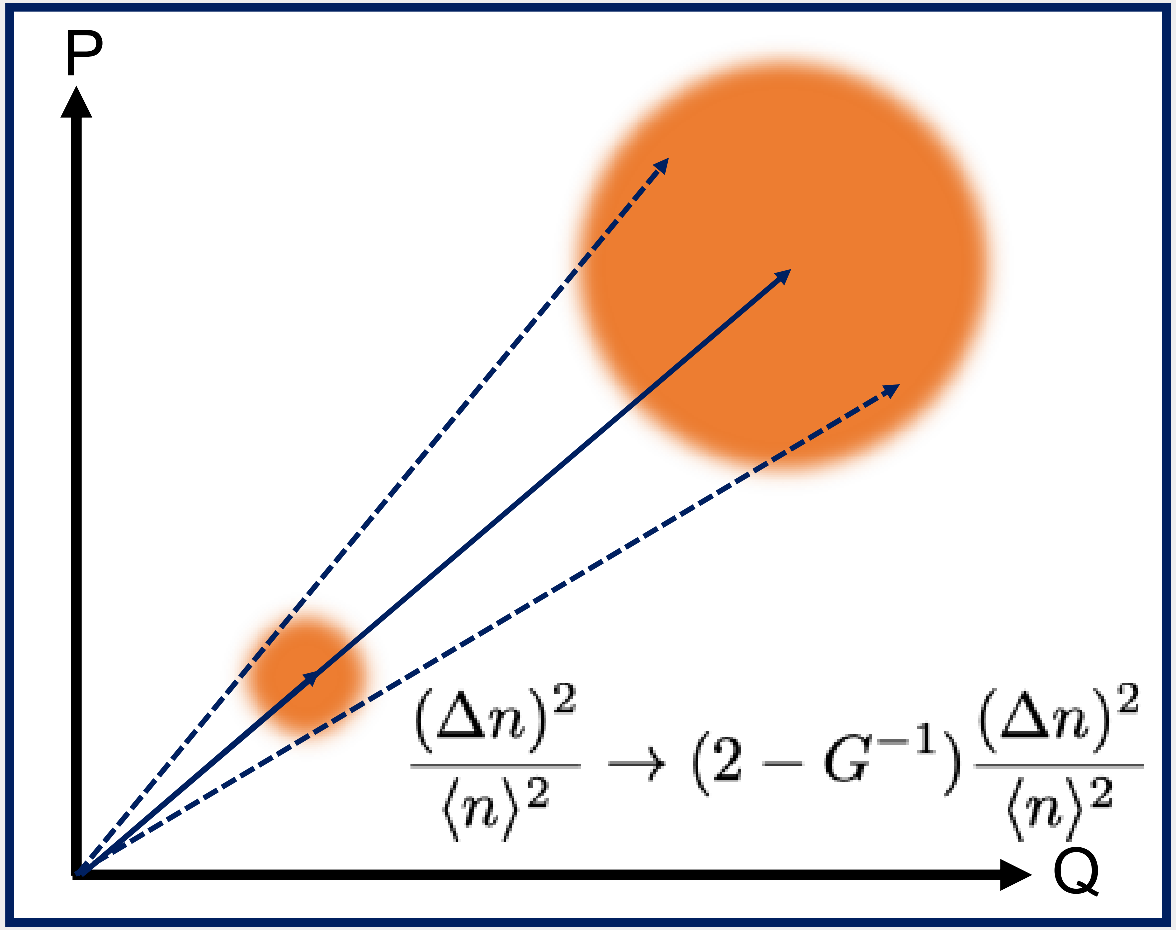
This noise limits almost all measurements based on low fluxes of photons!

Fundamental noise cost of amplification

Amplifying a signal by stimulated emission:

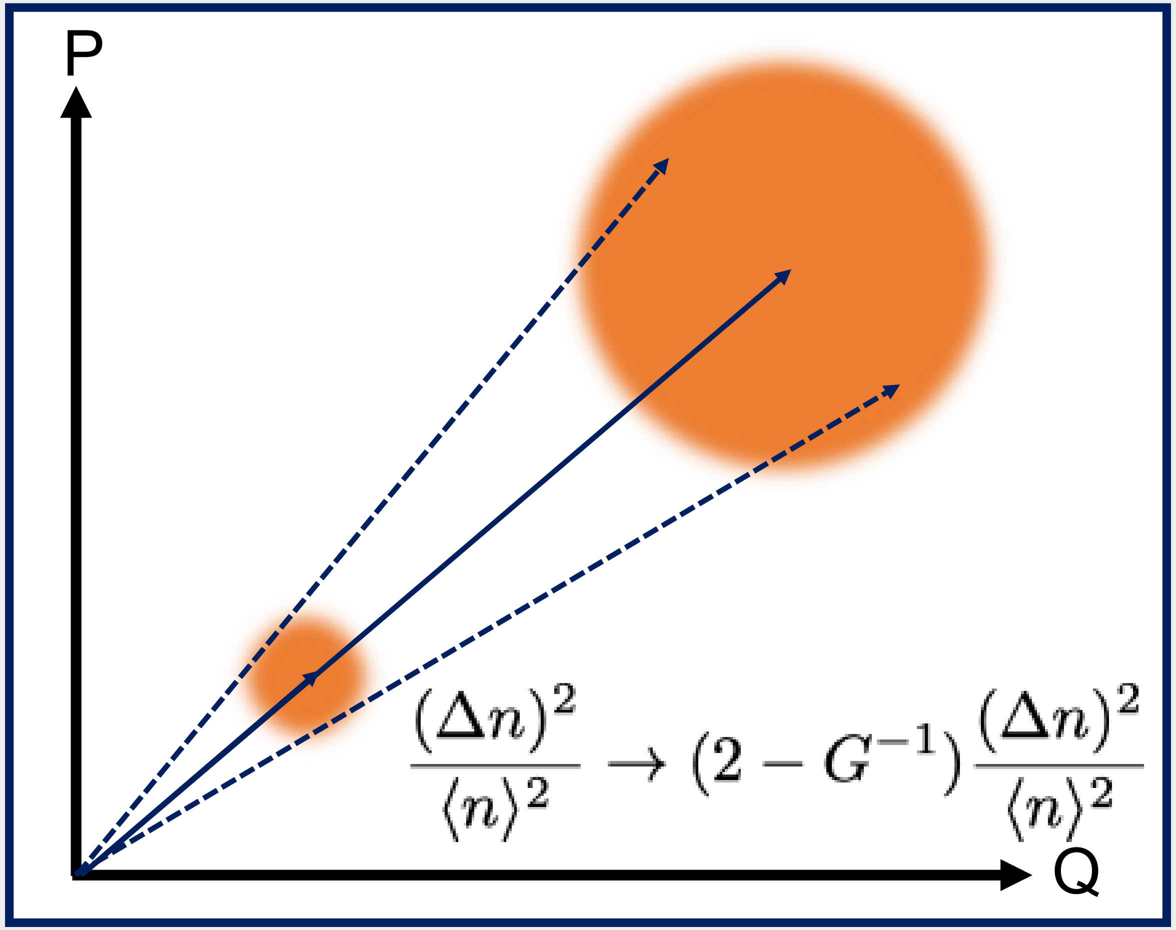


Fundamental noise cost of amplification



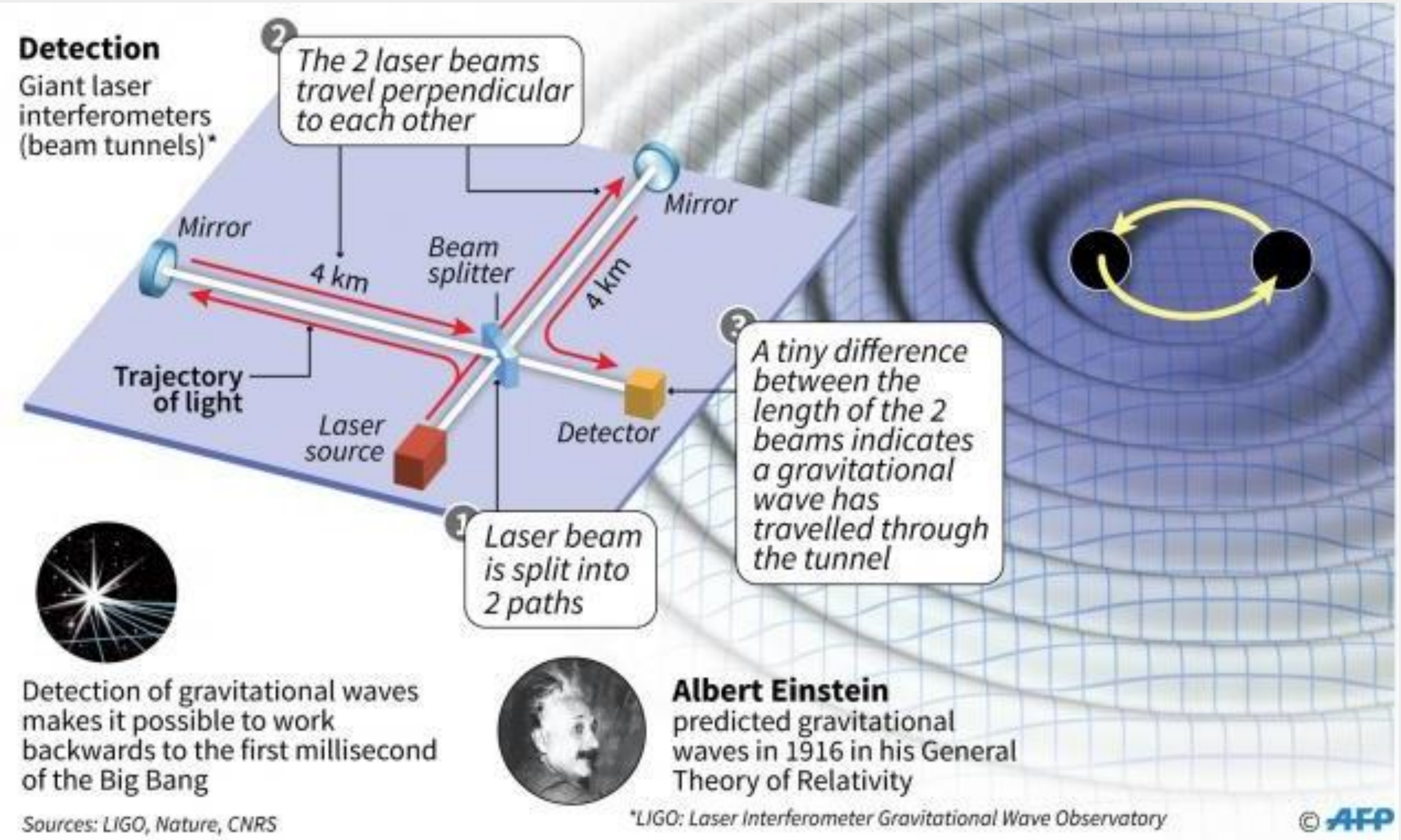
$$\frac{(\Delta n)^2}{\langle n \rangle^2} \rightarrow (2 - G^{-1}) \frac{(\Delta n)^2}{\langle n \rangle^2}$$

Fundamental noise cost of amplification



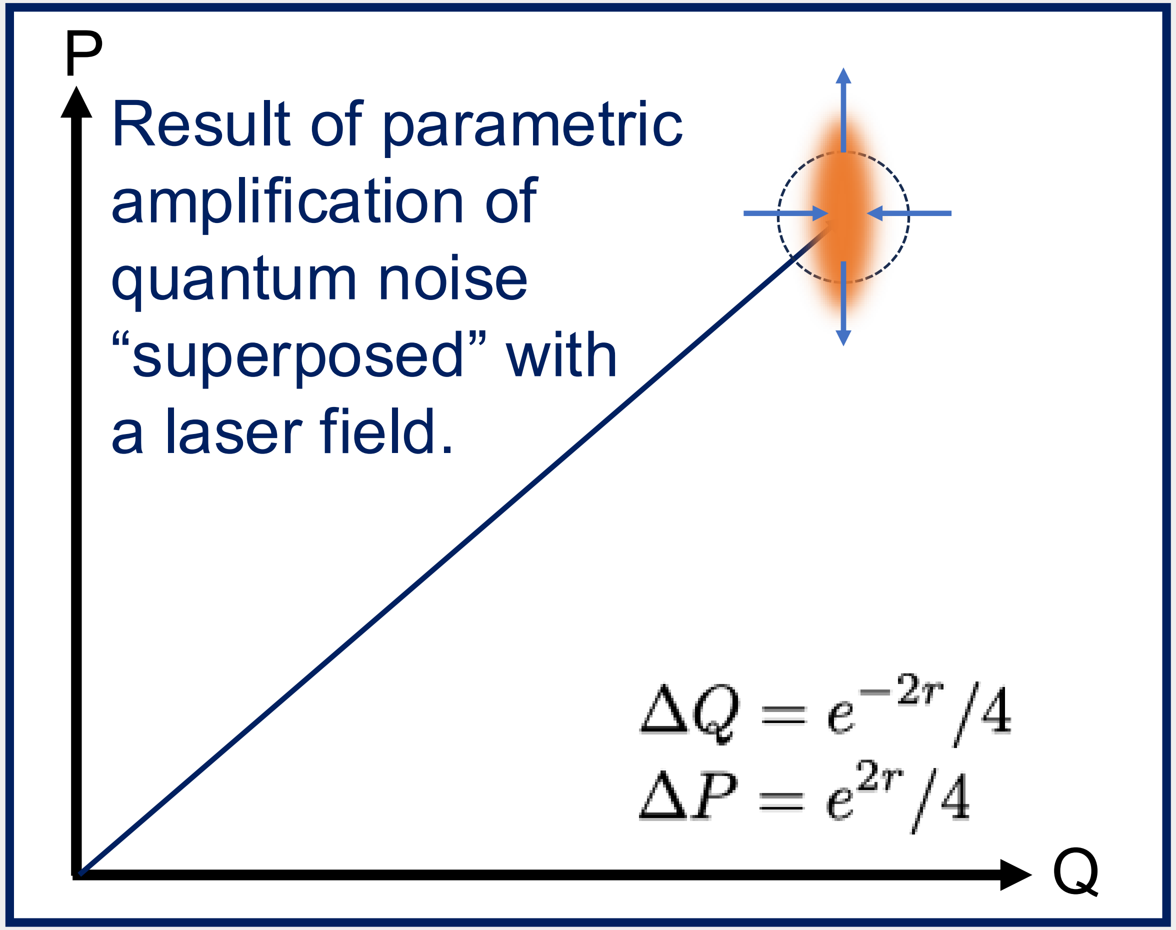
Amplifying light degrades its signal to noise ratio! Has major implications for the rate at which we can transmit data across long distances!

Detecting gravitational waves with light



Resolving weak signals buried in shot noise earned LIGO the 2017 Nobel Prize in Physics.

Reducing quantum noise by squeezing



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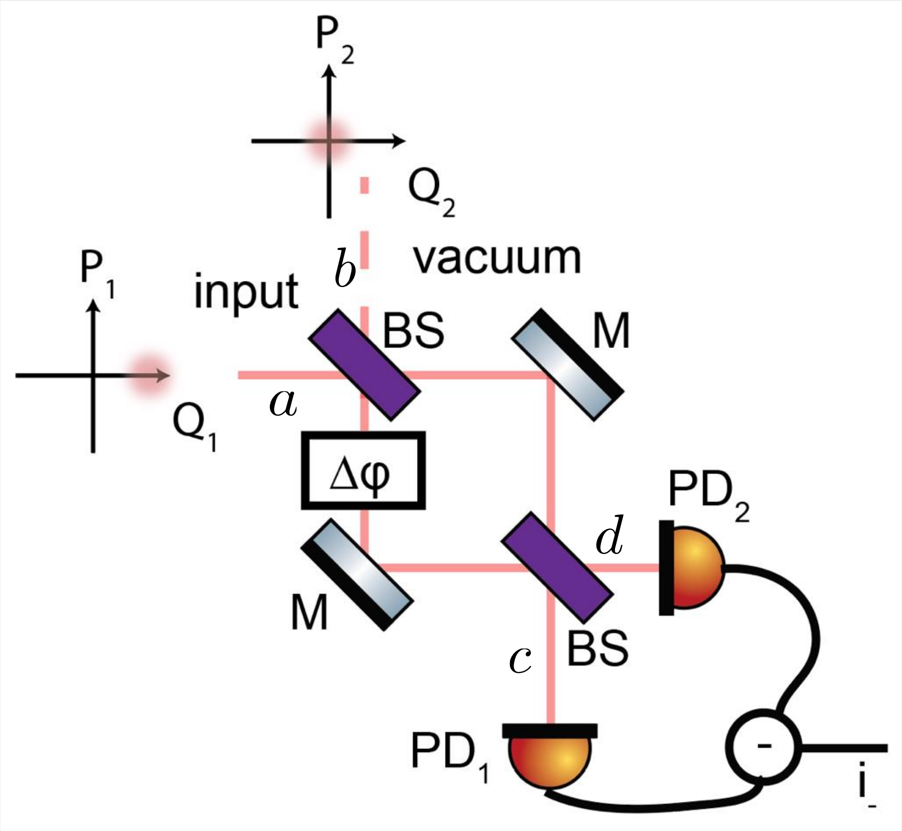
Shot noise limit in interferometry



$$i_- = c^\dagger c - d^\dagger d$$

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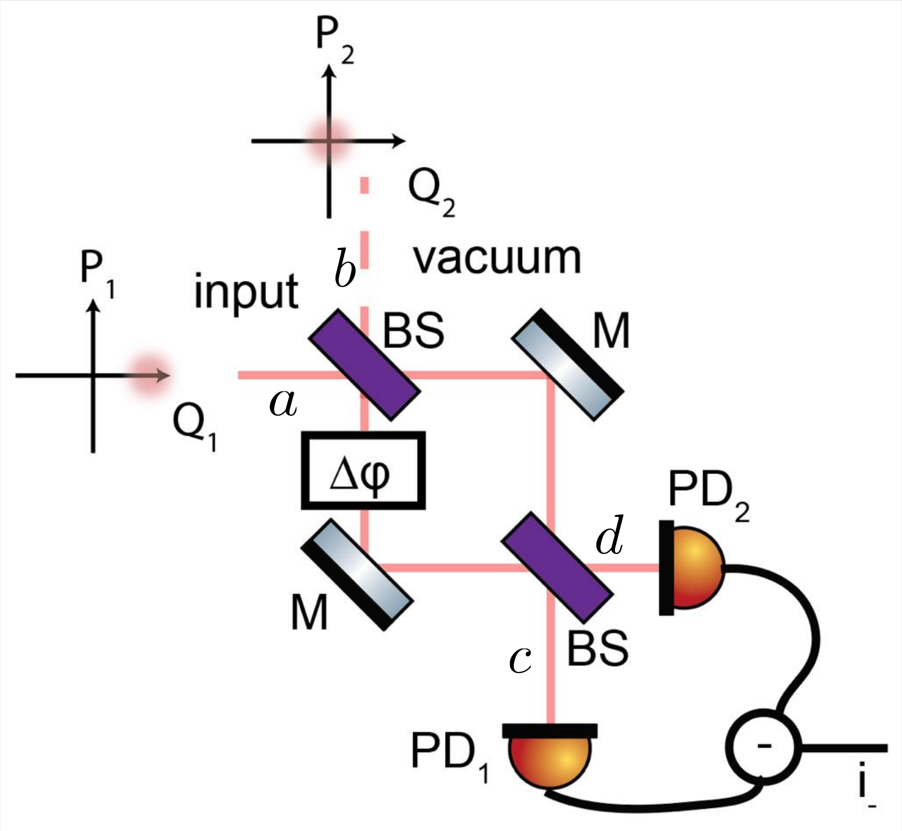

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$$(\Delta\phi)^2 \sim \frac{(\Delta Q_2)^2}{\langle n \rangle} \sim \frac{1}{\langle n \rangle}$$

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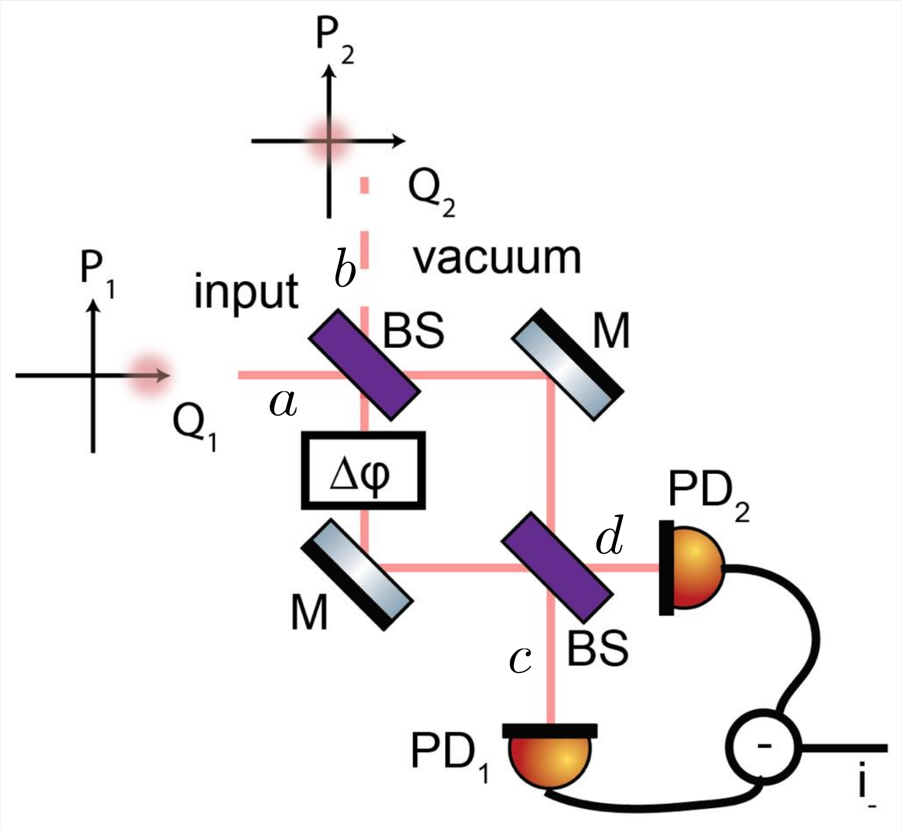

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$$(\Delta\phi)^2 \sim \frac{(\Delta Q_2)^2}{\langle n \rangle} \sim \frac{1}{\langle n \rangle}$$

The minimum detectable phase is related to the inverse number of photons times the vacuum fluctuations in the second input (“b”) port! This is the **shot noise limit.**

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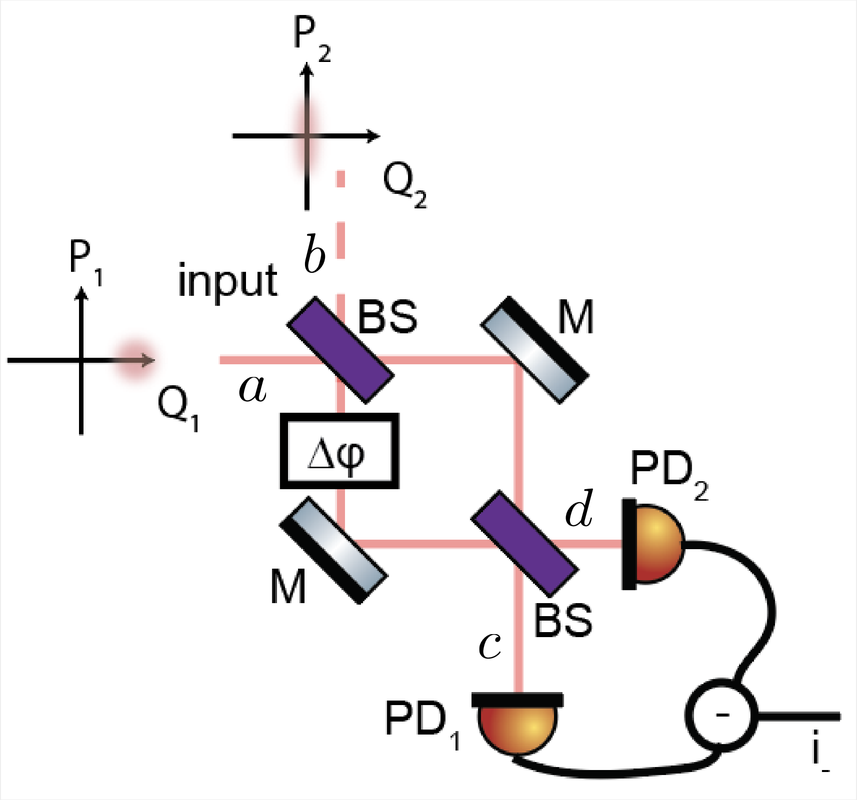
Quantum light interferometry



$$i_- = c^\dagger c - d^\dagger d$$

$$(\Delta\phi)^2 \sim \frac{(\Delta Q_2)^2}{\langle n \rangle} \sim \frac{e^{-2r}}{\langle n \rangle}$$

Sensitivity increases because the quantum noise in the "*b*" port has been compressed (core principle of the field of *quantum metrology*).

Impact of nonlinear & quantum optics

Nonlinear and quantum optics is tied to many Nobel Prizes*, including:

- Raman (Physics, 1930) – **Raman effect**
- Townes, Basov, Prokhorov (Physics, 1964) – principle of laser/maser
- Kastler (Physics, 1965) – optical pumping of two-level systems
- Gabor (Physics, 1971) – **holography**
- Bloembergen and Schawlow (Physics, 1981) – **nonlinear optics and laser spectroscopy**
- Glauber, Hall and Hansch (Physics, 2005) – **theory of quantum optics** & **frequency combs**
- Kao (Physics, 2009) – **optical fiber communications**
- Haroche and Wineland (Physics, 2012) – cavity quantum electrodynamics
- Akasaki, Amano, Nakamura (Physics, 2014) – Blue LED
- Betzig, Hall, Moerner (Chemistry, 2014) – Super-resolution microscopy
- Weiss, Thorne, Barish (Physics, 2017) – **gravitational wave detection**
- Ashkin, Mourou and Strickland (Physics, 2018) – optical trapping and **chirped pulse amplification**
- Aspect, Clauser, Zeilinger (Physics, 2022) – **entanglement of photons**
- Agostini, Krausz, L'Hullier (Physics, 2023) – **high-harmonic generation and attosecond pulses**

**Not the only measure of impact, but it makes a statement!*

On Monday: introducing the nonlinear susceptibility and its microscopic origin