Spatial noise dynamics in nonlinear multimode fibers

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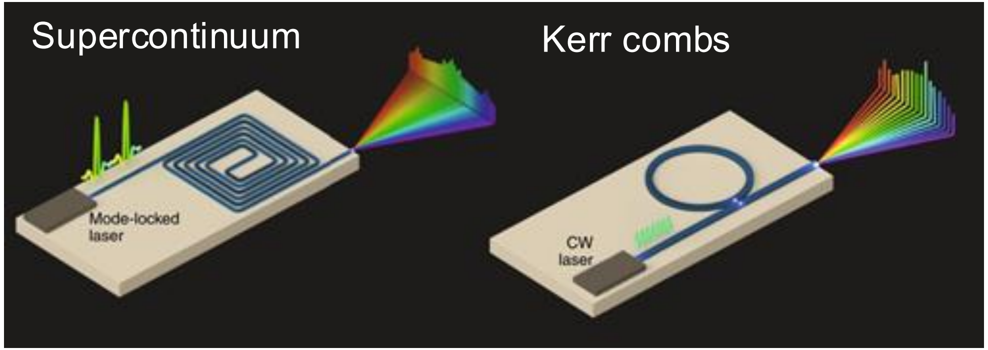
May 9, 2025

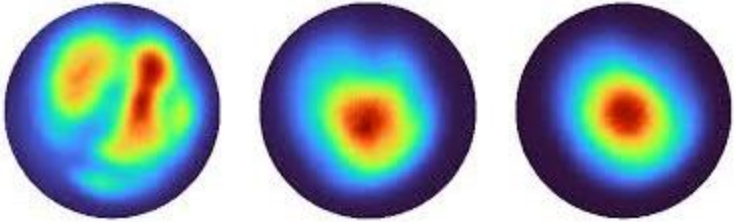
In this talk:

- Importance of noise in nonlinear multimode photonics
- Framework for calculating noise in generic nonlinear multimode systems
- Experimental probe of noise in multimode fibers
- Optimization of noise via wavefront shaping

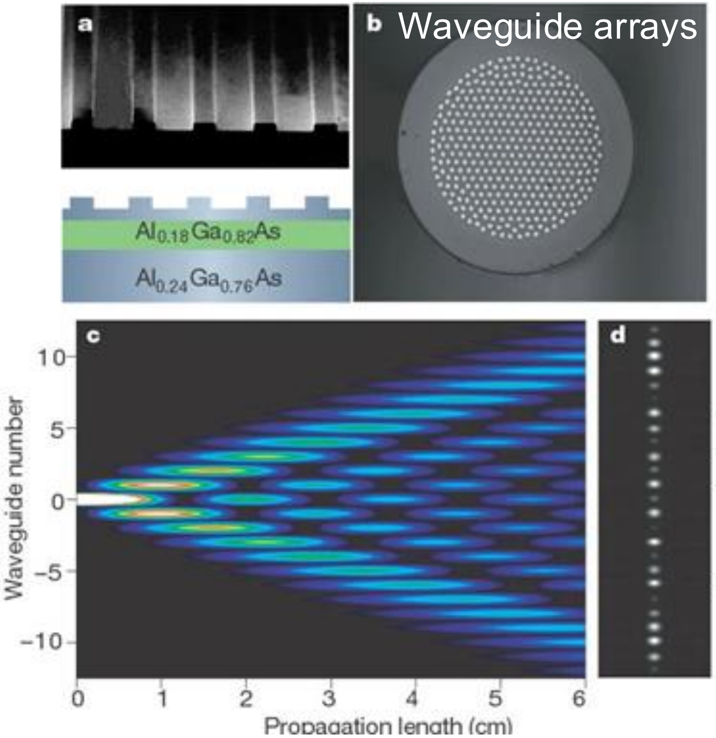
Multimode nonlinear systems



Gaeta, Lipson, Kippenberg
Nat. Photonics (2019)



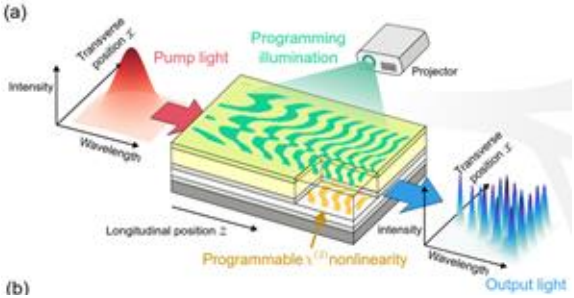
Wright et al.
Science (2017)



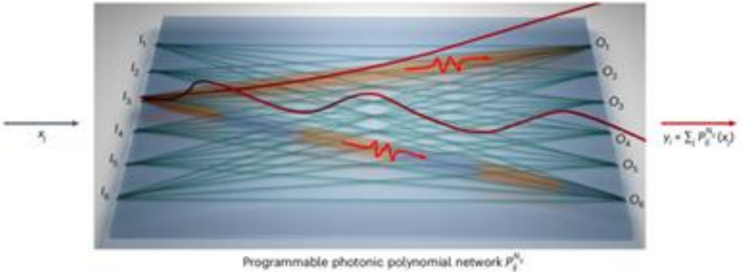
Christodoulides et al.
Nature (2003)

Increasingly programmable

Programmable nonlinear photonics



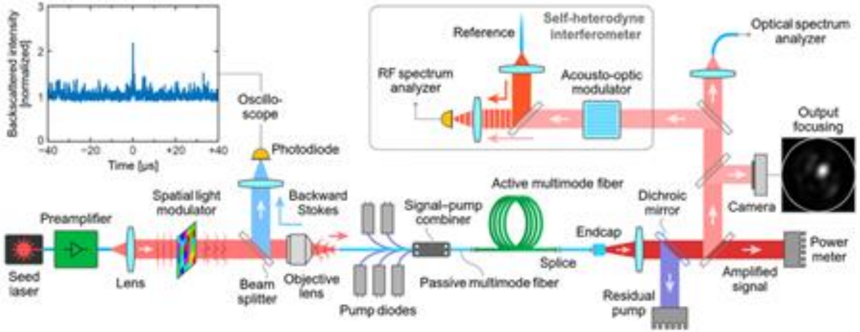
Yanagimoto et al
arXiv:2503.19861 (2025)



Wu et al
Nat. Photonics (2025)

Improved multimode fiber lasers

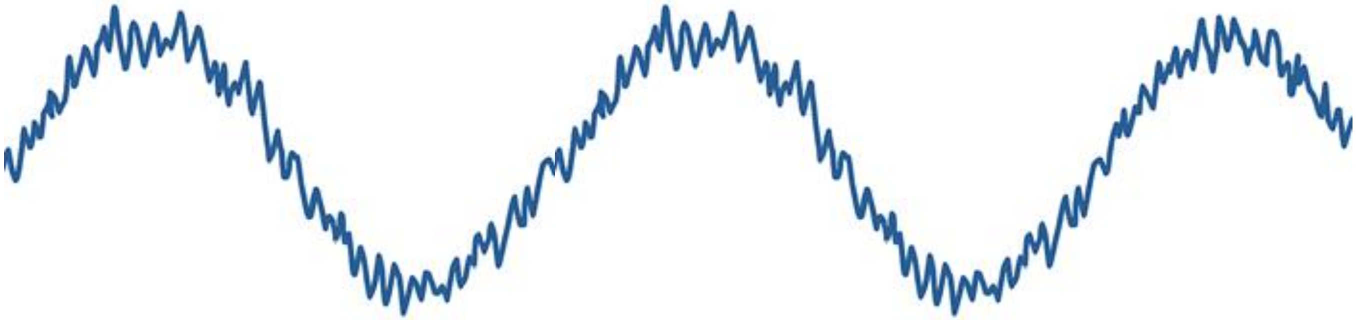
Rothe et al. arxiv:2504.06423 (2025)



Importance of noise in optics

Many applications limited by **noise**:

- Linewidth and stability of oscillators
- Data transmission
- Optical computing
- Quantum metrology and computing



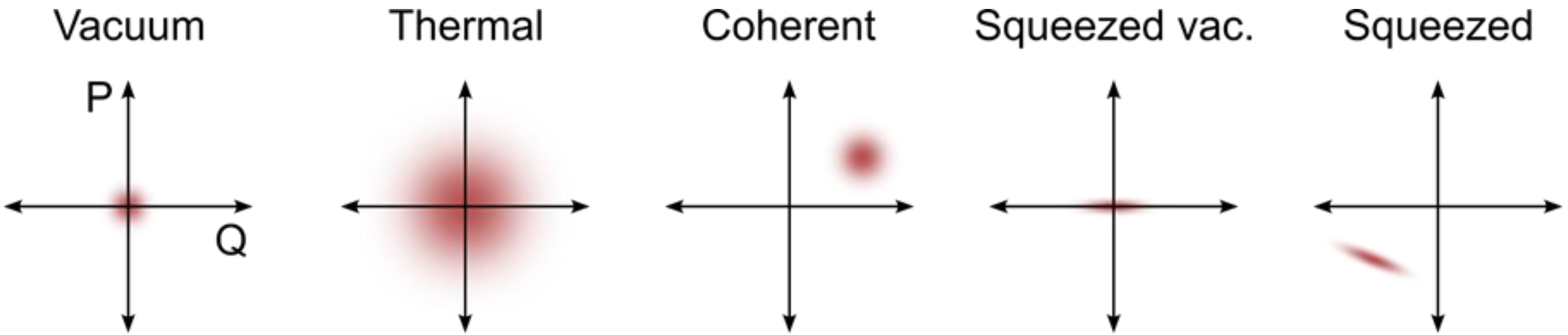
Single mode gaussian states

$$[a, a^\dagger] = 1$$

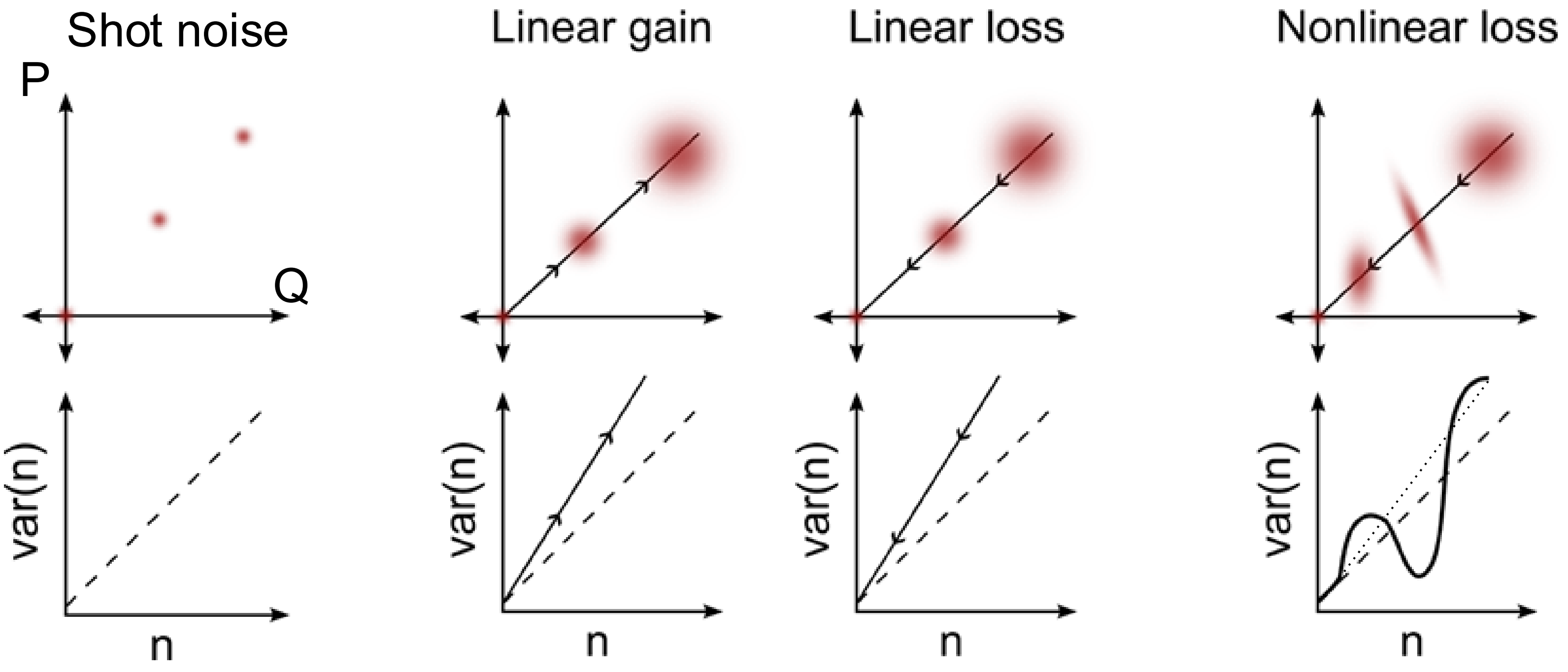
Quantized EM field

$$a = \langle a \rangle + \delta a$$

Mean field   Noise



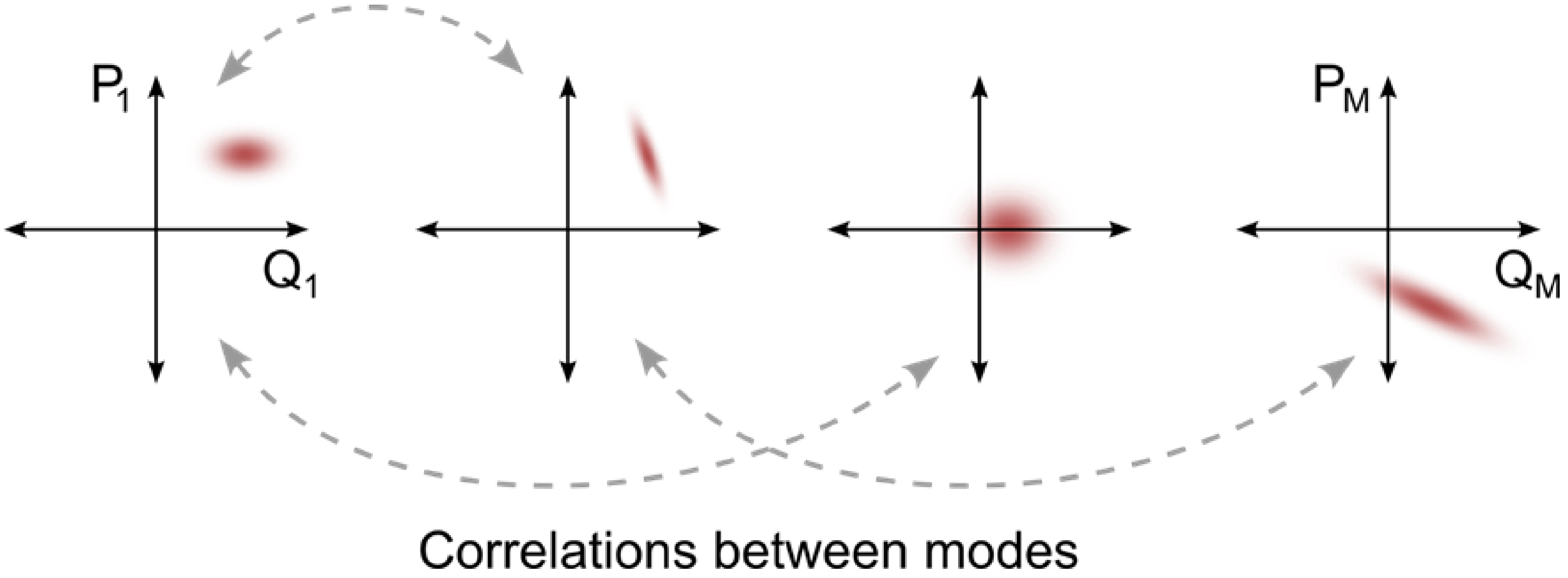
Noise behavior of gaussian states



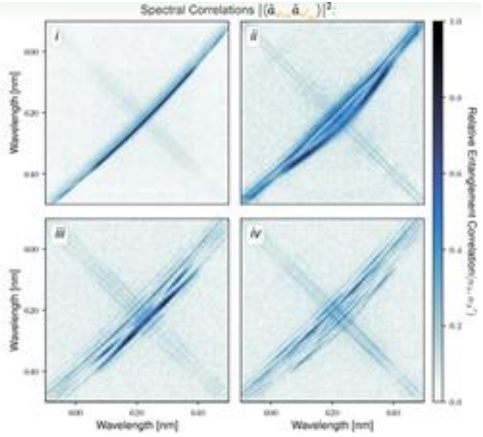
Nonlinear attenuation can beat linear equivalent

Multimode gaussian states

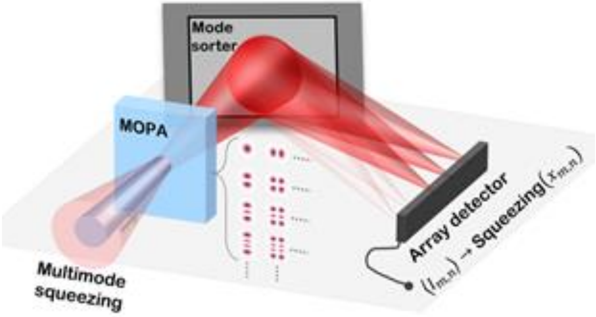
$$[a_i, a_j^\dagger] = \delta_{ij}$$



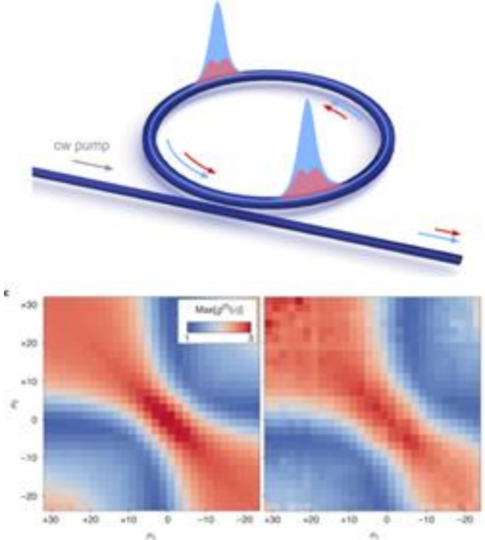
Multimode quantum nonlinear optics



Stein et al
arXiv:2401.06119 (2024)



Kalash et al
arXiv:2503.07486 (2025)



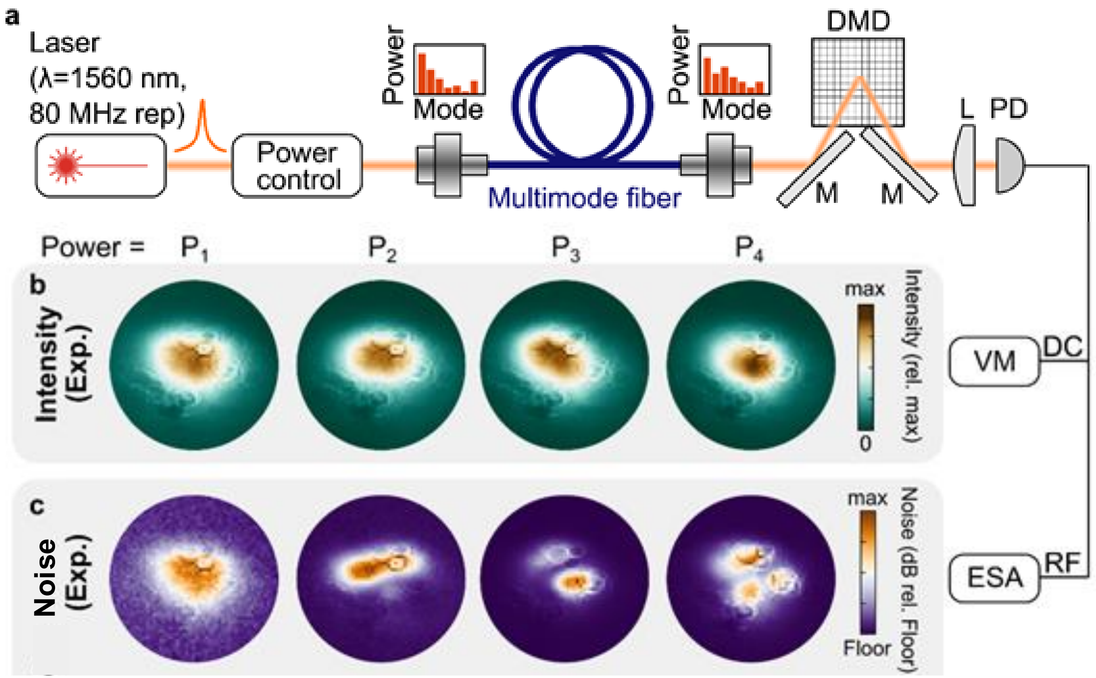
Guidry et al, Nature Photonics (2022)
Lustig, Guidry et al (2024)

It would be great to:

- Control noise in multimode nonlinear systems
- Create high powered, low noise states
- Beat the rules of linear attenuation
- Controllably create multimode quantum states
- Find squeezing hidden in plain sight

An experiment to probe multimode noise

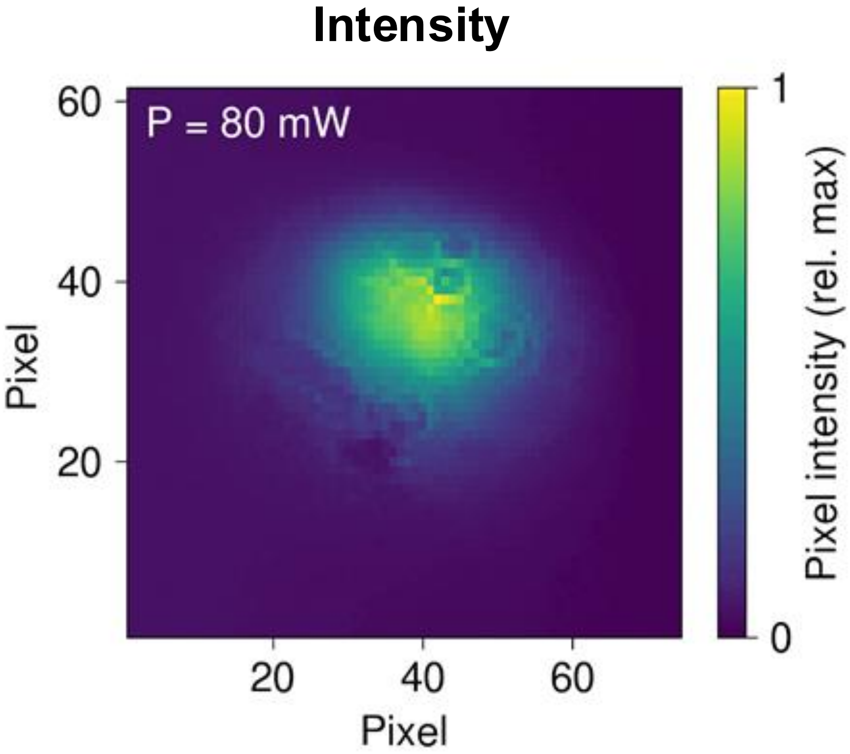
Experimentally probing multimode noise

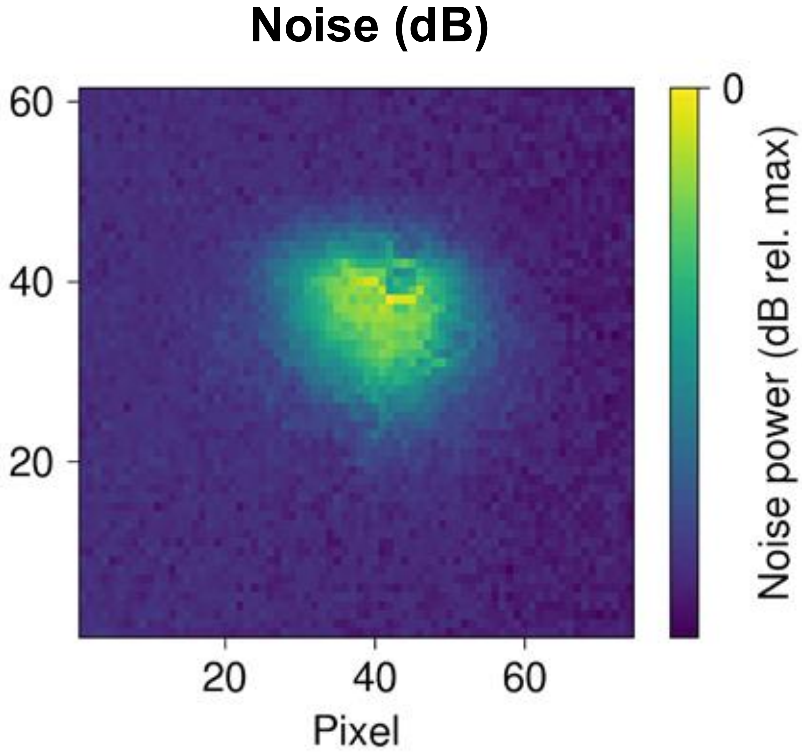


A few observations:

- Nonlinearity causes power dependence of intensity and noise
- Noise can look totally different than the intensity

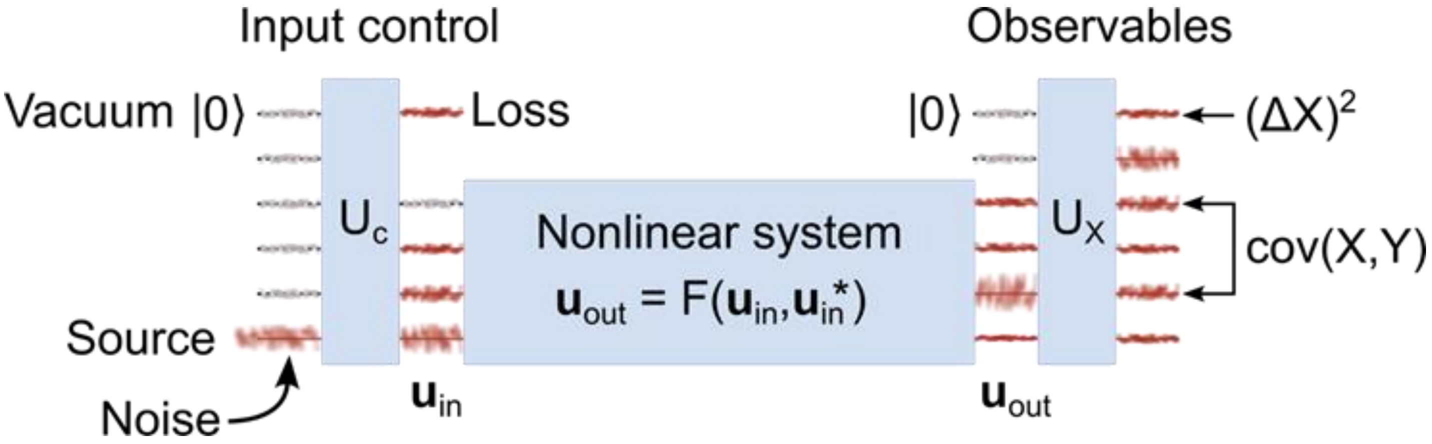
Power dependent intensity and noise map





A framework for multimode noise

Noise in multimode photonic systems



$$\delta\hat{X} = \sum_k \frac{\partial X}{\partial u_{\mathrm{in},k}} \delta\hat{u}_{\mathrm{in},k} + \frac{\partial X}{\partial u^*_{\mathrm{in},k}} \delta\hat{u}^\dagger_{\mathrm{in},k}$$

“Quantum sensitivity analysis”

Quantum sensitivity analysis

Variance in any observable:

$$(\Delta X)^2 = \sum_k \left| \frac{\partial X}{\partial u_{\mathrm{in},k}} \right|^2 F_{\mathrm{in},k}$$

Sum over all channels	Classical gradients	Input noise

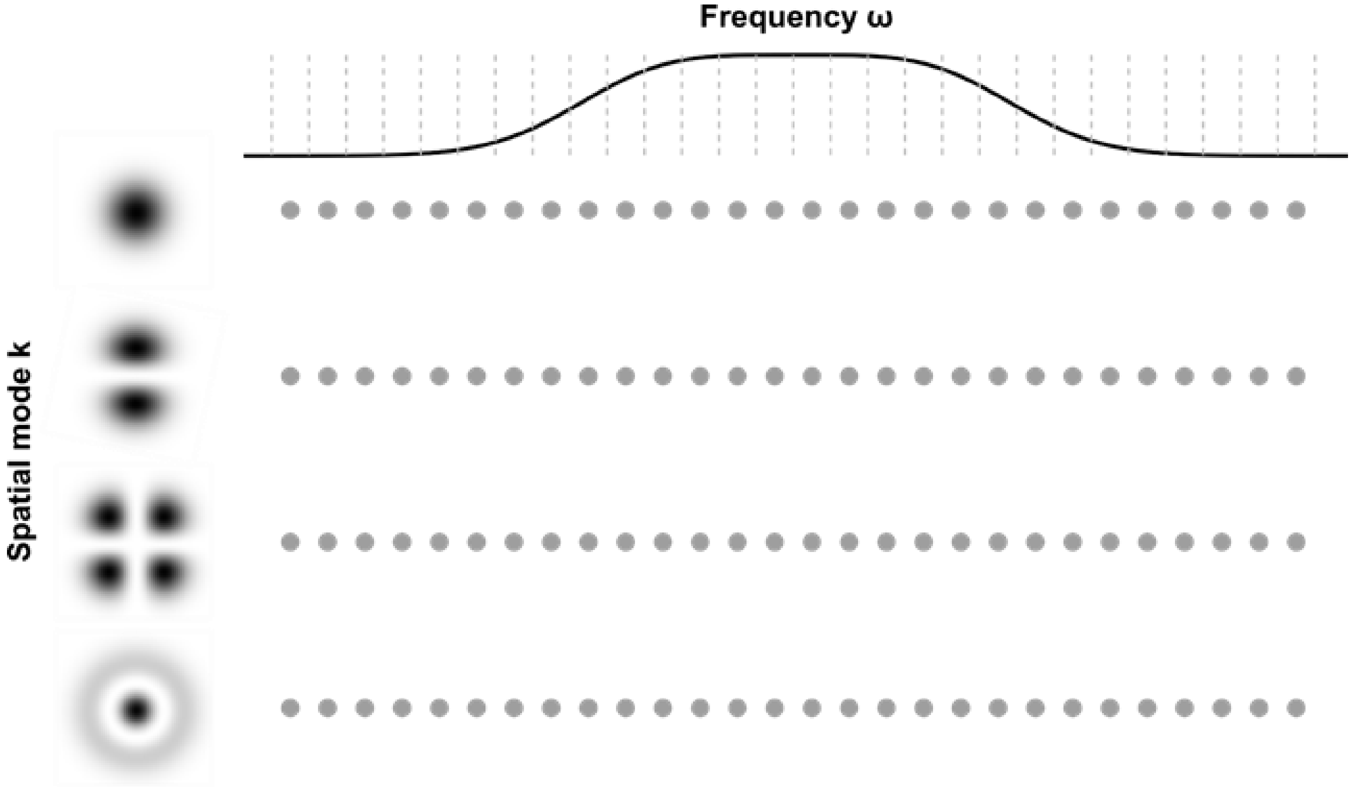
Advantages:

- Classical simulations only
- Single simulation, any input noise
- Physically insightful

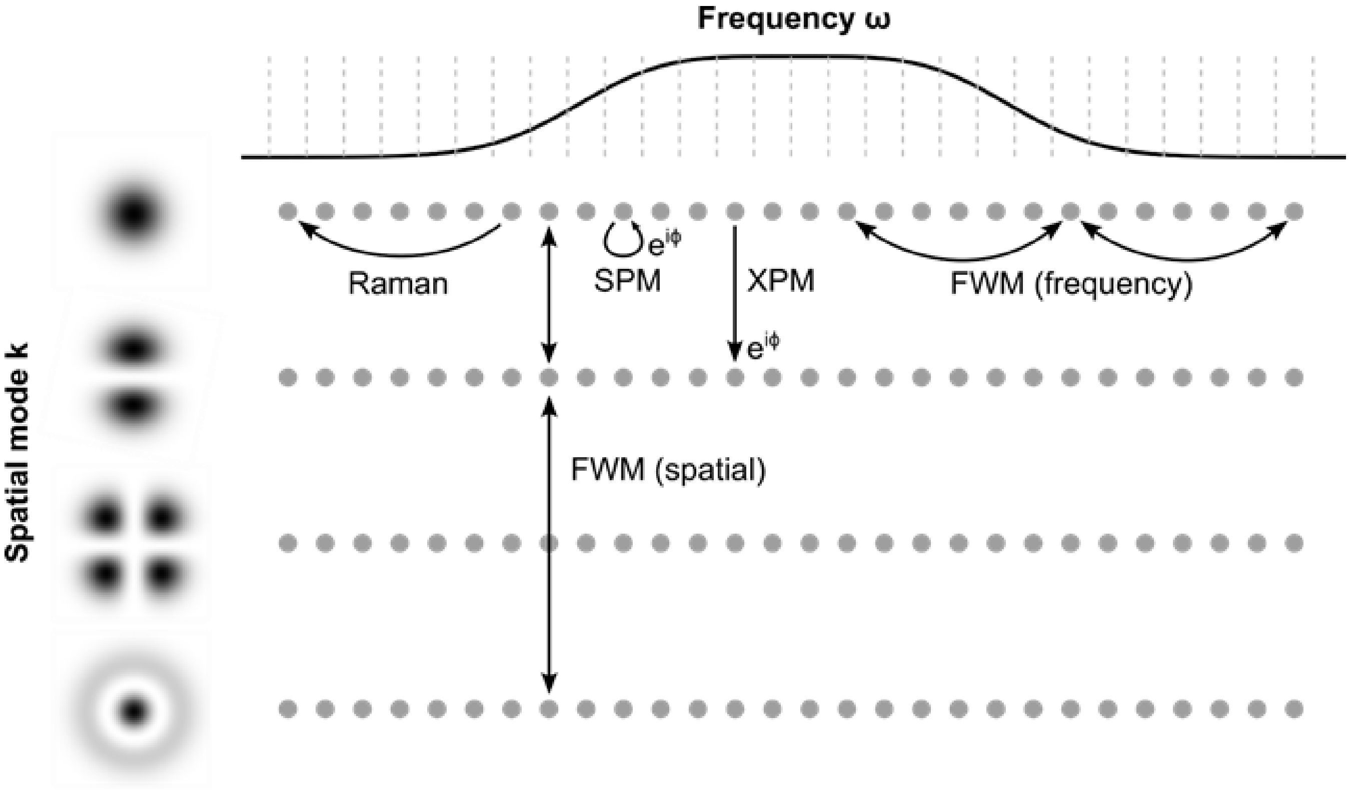
"Ultra-broadband and passive stabilization of ultrafast light sources by quantum light injection"
Rivera, Uddin, **Sloan**, Soljacic. *Nanophotonics* (2025)

"Noise-immune quantum correlations of intense light"
Uddin*, Rivera*, Syler, **Sloan**, …, Soljacic. *Nature Photonics* (In press)

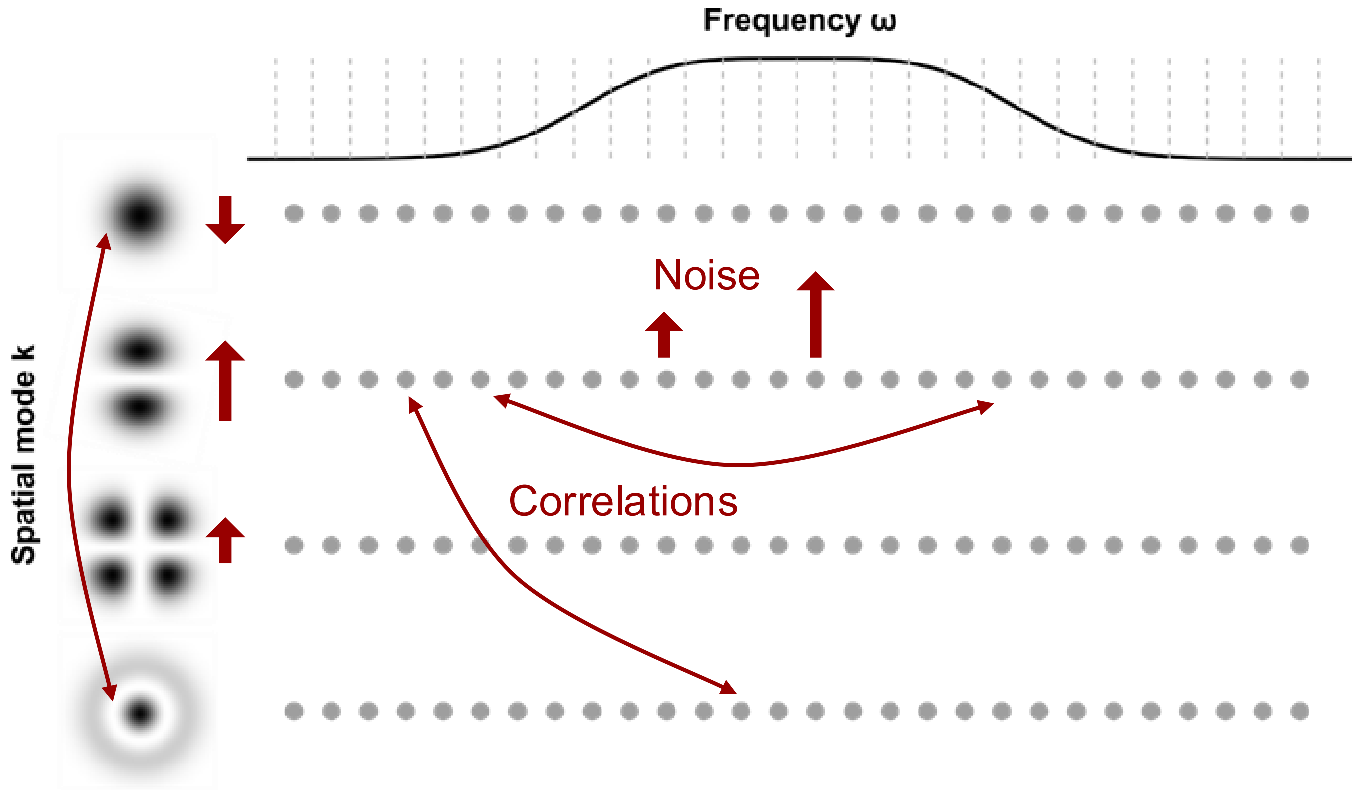
Nonlinear propagation in multimode fibers



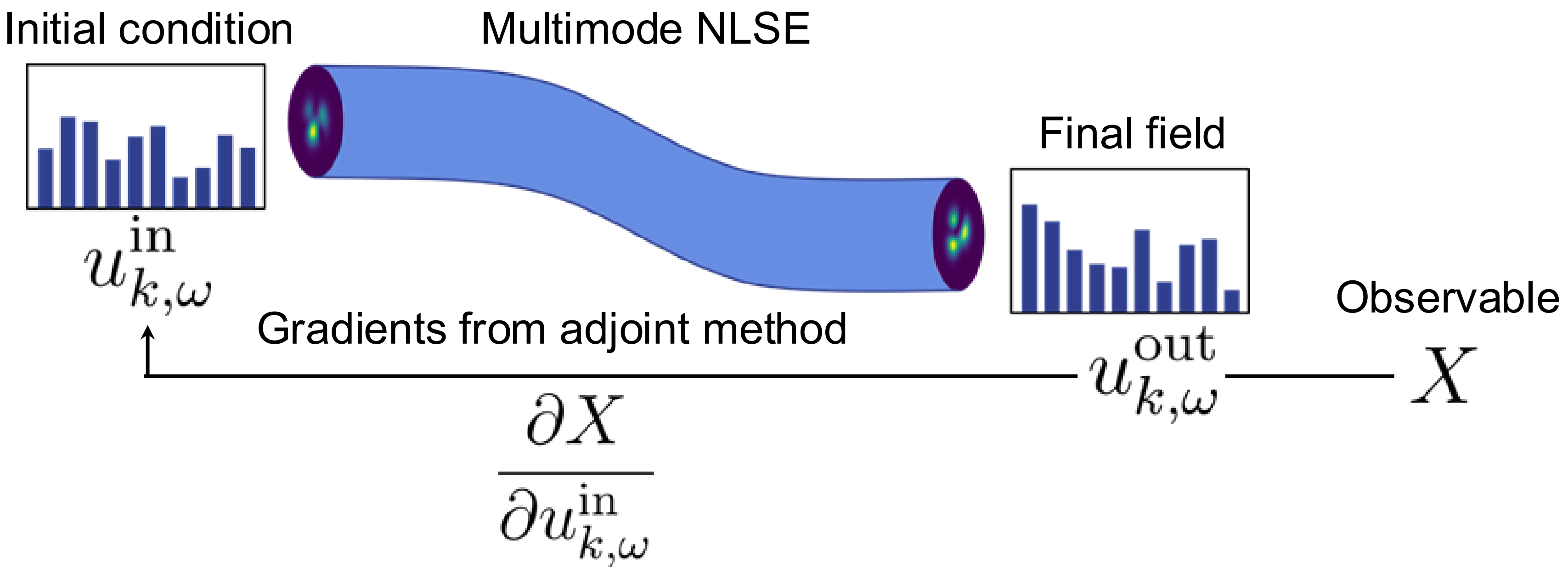
Nonlinear propagation in multimode fibers



Nonlinear propagation in multimode fibers



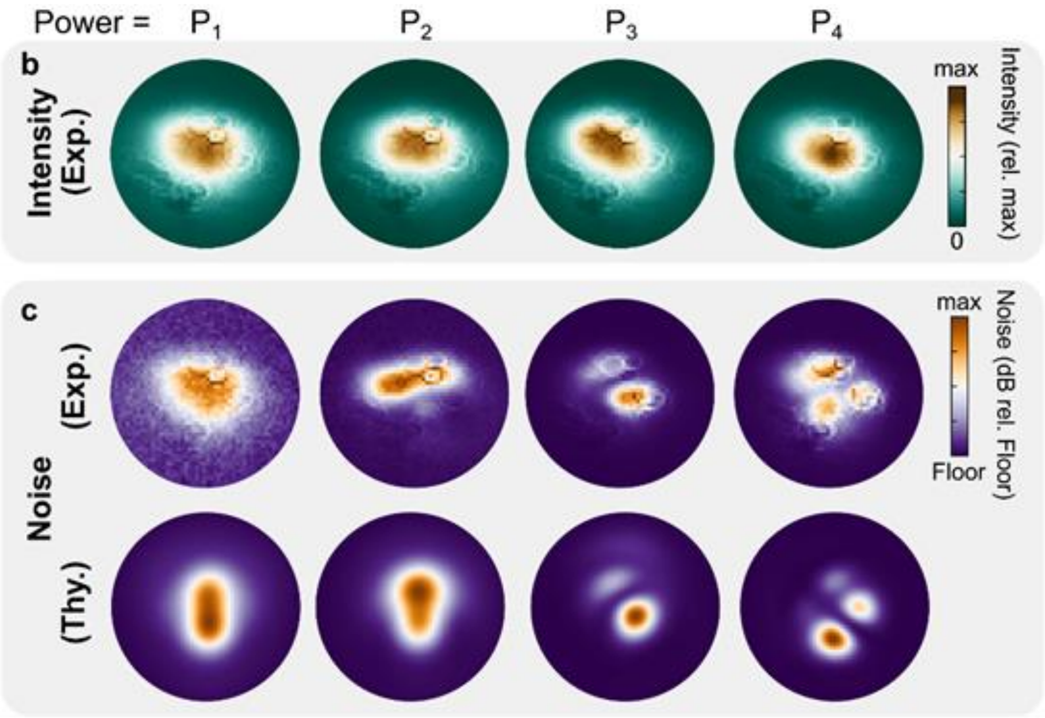
Quantum sensitivity analysis on MMFs



Efficient prediction of

- Noise level of fiber modes, or spatial pixels
- Correlations between modes or pixels
- Response to different input noise or correlations

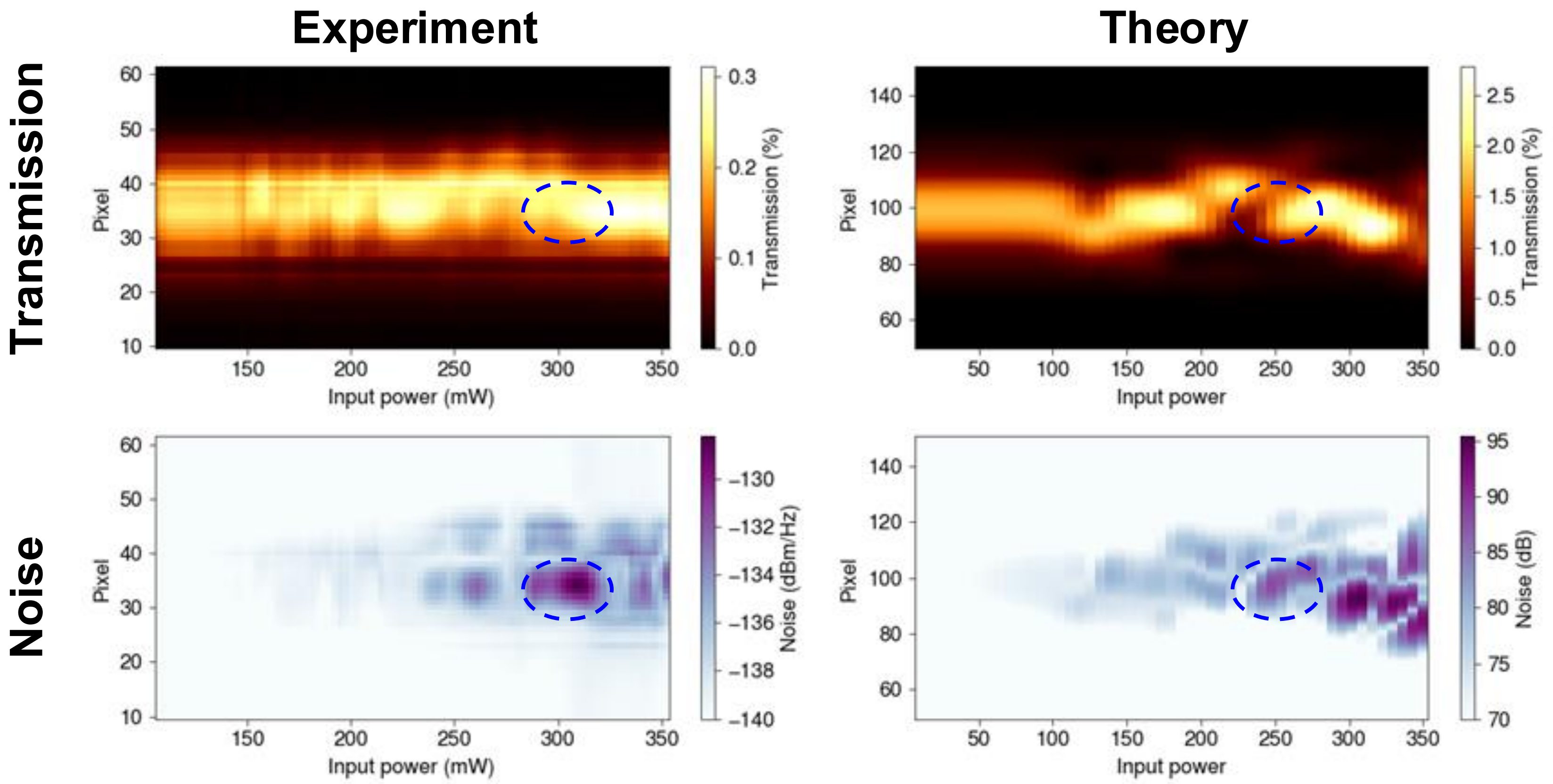
Theory vs experiment images



Theory tells us that:

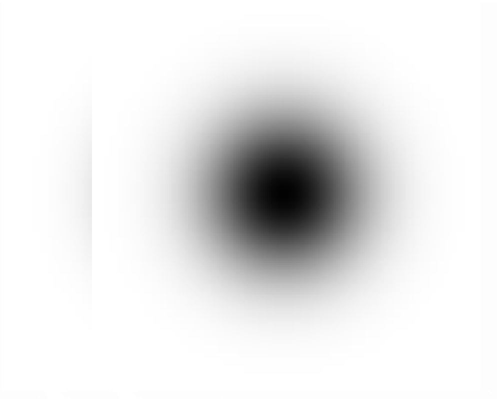
- Higher order modes can become very noisy
- Amplified laser noise often dominates
- Shot noise important at low noise points

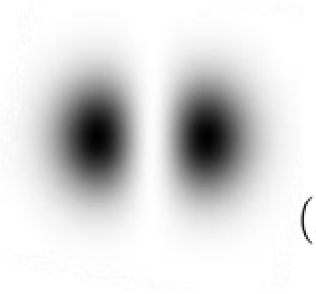
Noise behavior with nonlinear transmission

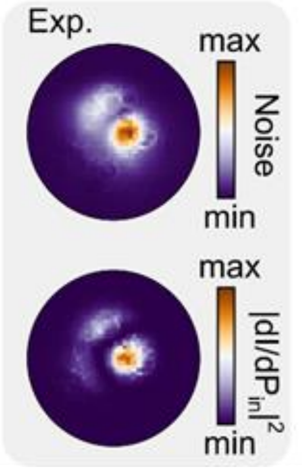


Transmission changes → noise changes

Noise can sense input power fluctuations







$$(\Delta I)^2 \sim \left(\frac{dI(x,y)}{dP_{\text{in}}} \right)^2$$

Occurs when:

- Source laser is noisy
- Output observables are only sensitive to the total input power

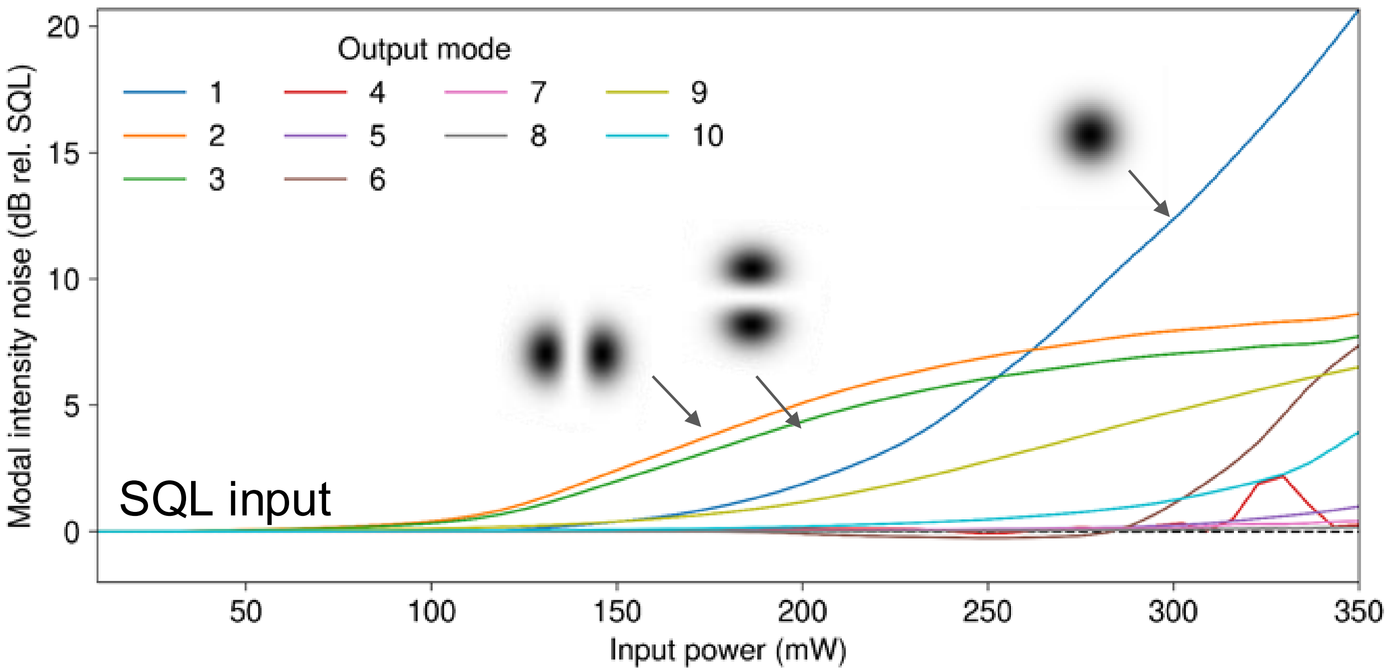
Fails when:

- Shot noise is important
- Output responds differently to noise at different frequencies

New insights from theory

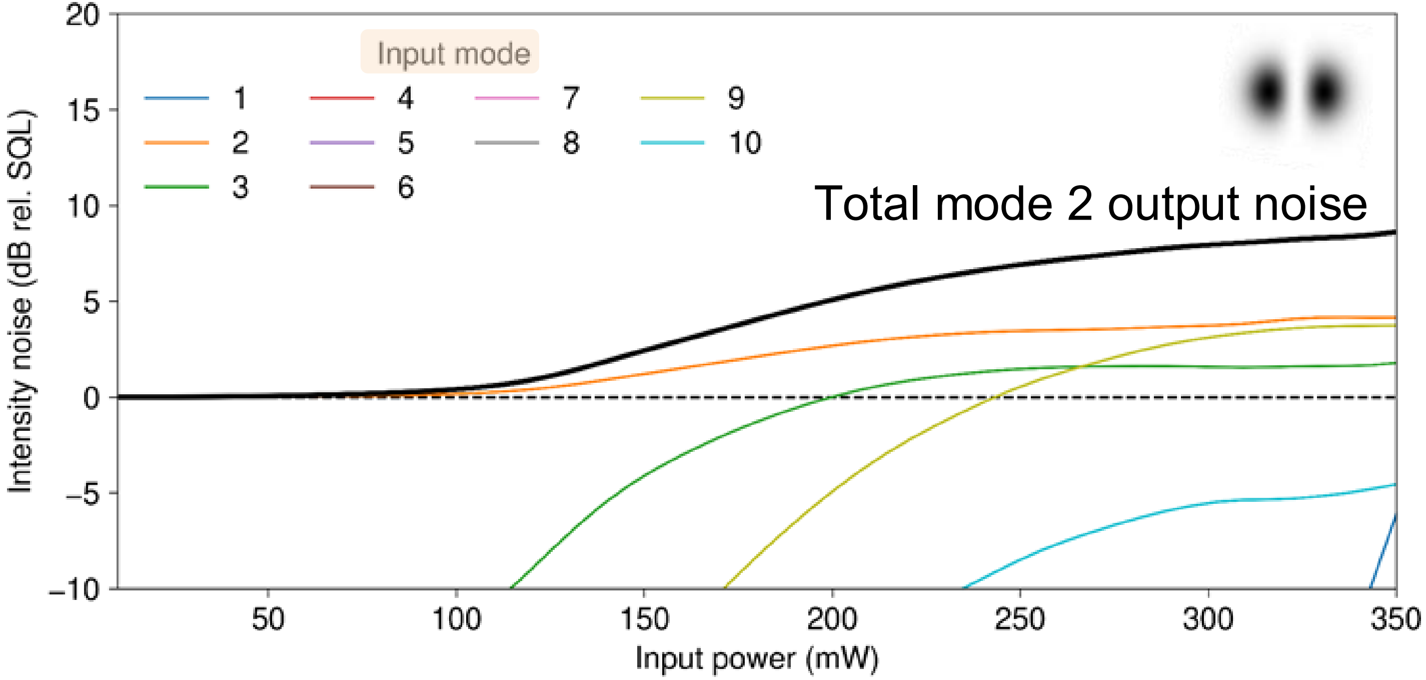
Modal noise from theory

$(\Delta I_l)^2$



Many modes pick up excess intensity noise!

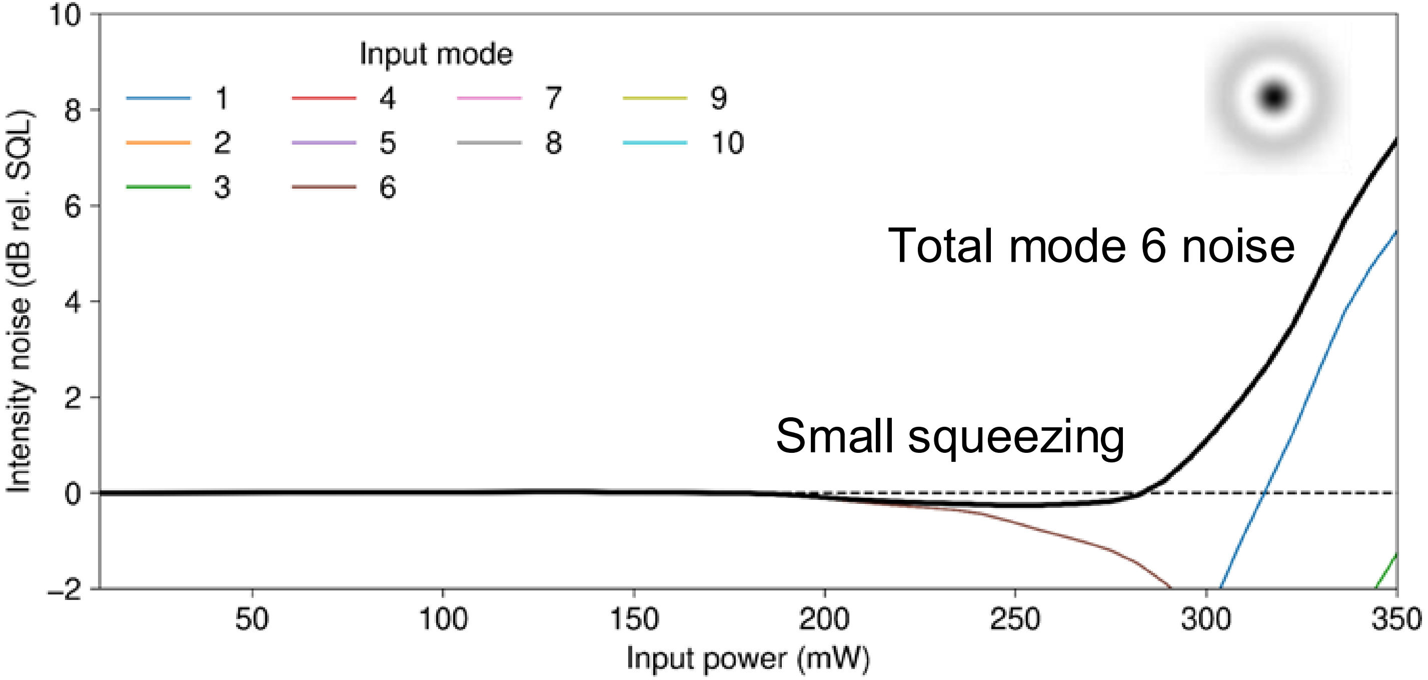
Modal noise from theory



Multiple sources

$$(\Delta I_l)^2 = \sum_{k,\omega} \left| \frac{\partial I_l}{\partial u^{\mathrm{in}}_{k,\omega}} \right|^2$$

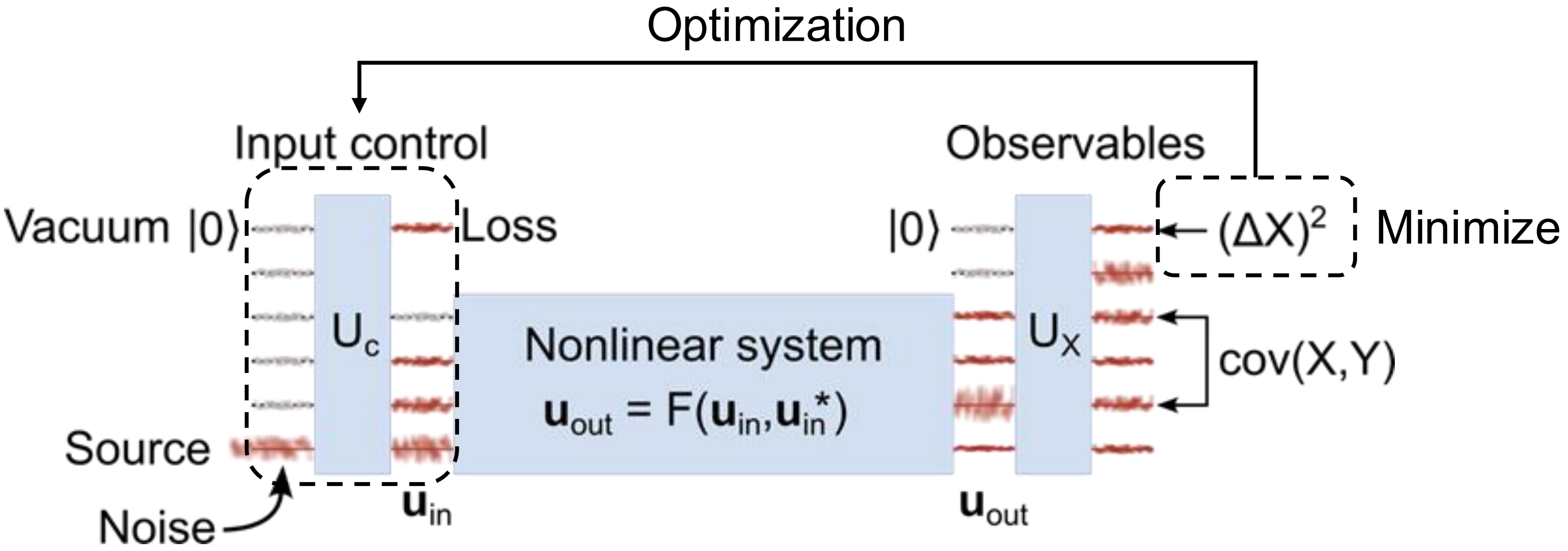
Squeezing of intense spatial modes



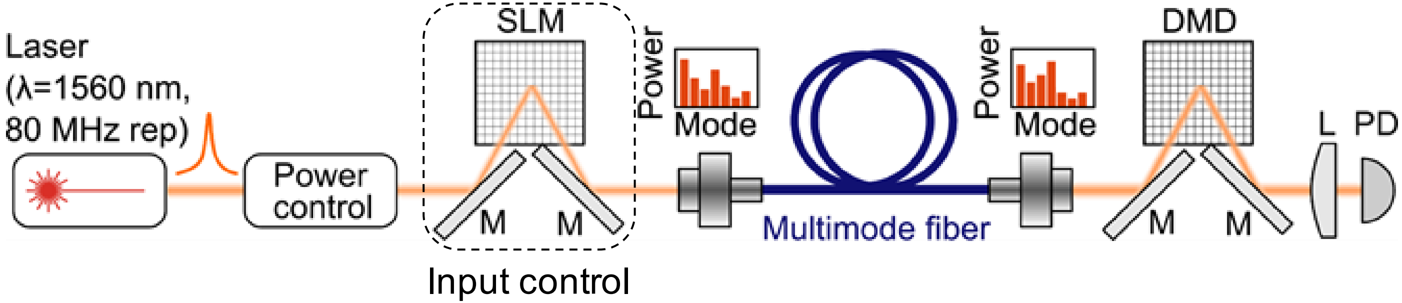
$$(\Delta I_l)^2 = \sum_{k,\omega} \left| \frac{\partial I_l}{\partial u_{k,\omega}^{\text{in}}} \right|^2$$

Gaining control over noise

Wavefront shaping to control noise (thy.)

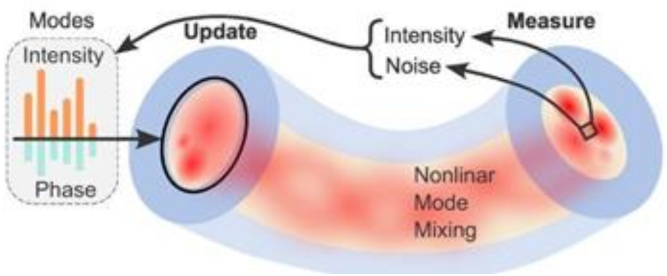


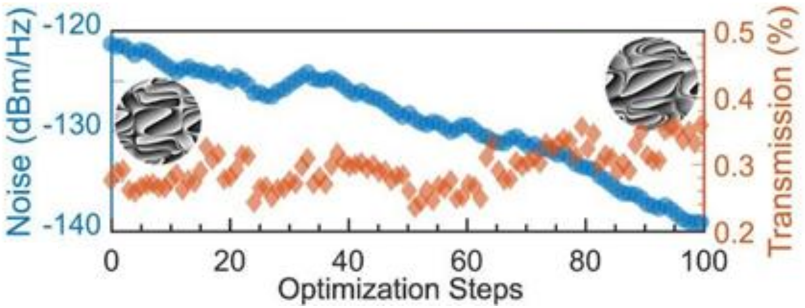
Experiment for shaping initial conditions



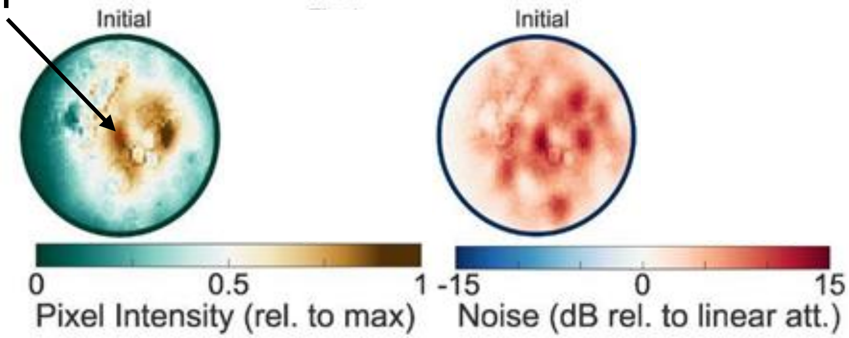
Shaping ICs widens the space of noise behaviors

Optimization on the experiment





Target pixel



— High intensity, low noise!

Key takeaways

- In nonlinear systems, the rules of how noise behaves can be rewritten

- In multimode systems, it's possible for some outputs to be resistant/immune to input noise

- Uncontrolled nonlinearity makes noise worse, but controlled nonlinearity makes it better

We hope this work can enable…

- Better high powered fiber lasers

- Intense squeezed sources

- Input shaping of nonlinear systems to produce desired output squeezed or correlated states

Acknowledgments

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MIT

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Harvard → Cornell

Marin Soljačić
MIT

Thanks for listening!

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Sensitivity analysis on fibers

$$(\Delta n(x,y))^2 = \bar{n}(x,y) + \left(\sum_{k=1}^{M} \sum_{\omega} \left| \frac{\partial n(x,y)}{\partial u_{k,\omega}^{\text{in}}} \right|^2 - \sum_{k=1}^{M} |\phi_k(x,y)|^2 \right)$$

$$+ \sum_{k=1}^{M} \sum_{\omega} \left| \sum_{l=1}^{M} U_l \frac{\partial n(x,y)}{\partial u_{l,\omega}^{\text{in}}} \right|^2 \delta F_{\omega}^{\text{in}}$$

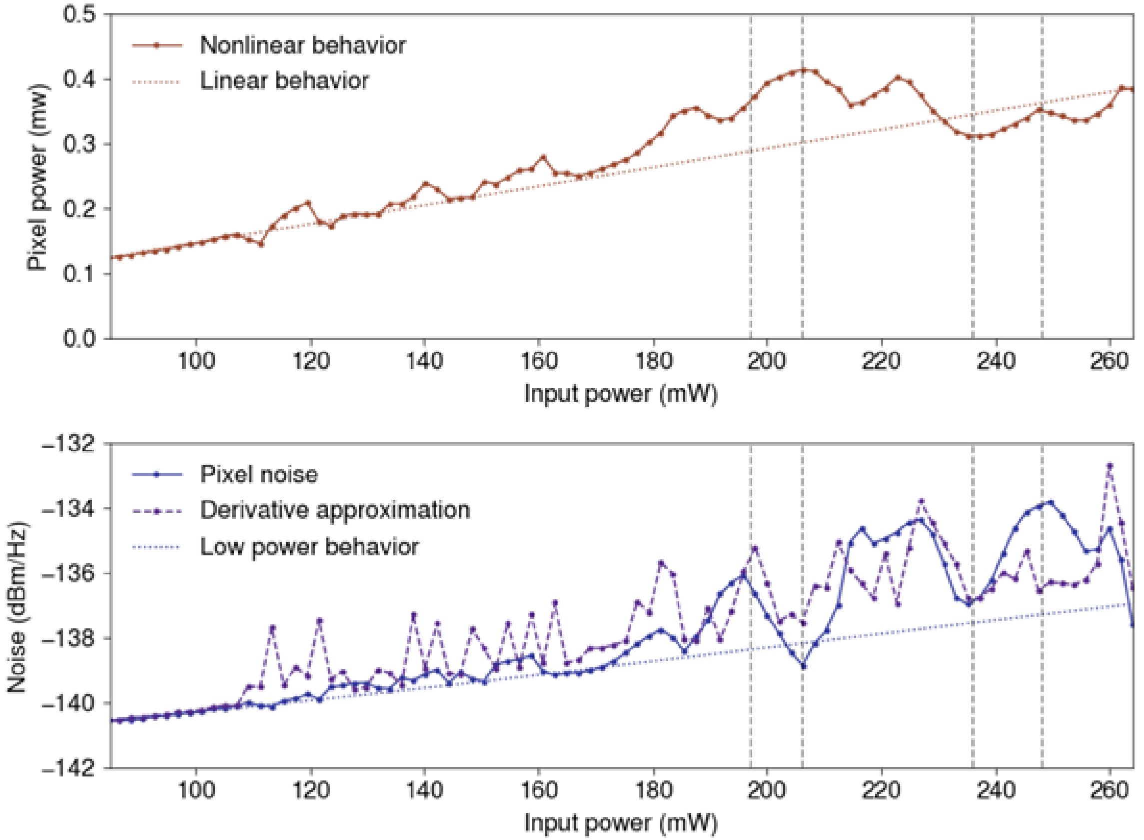
Shot noise behavior (incl. Vacuum modes!)

Fiber initial condition

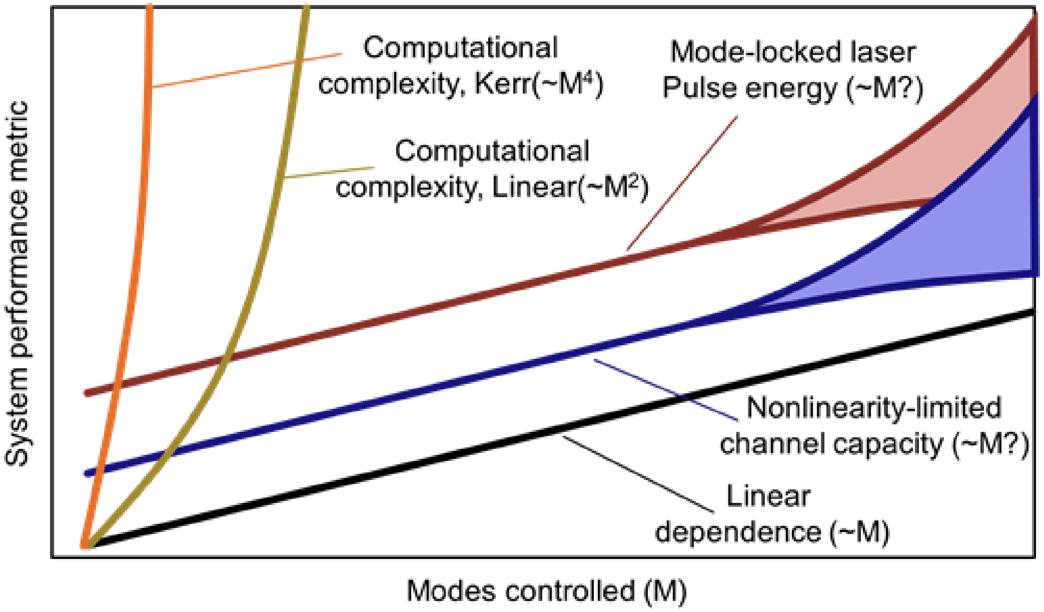
Shot noise

Behavior of excess laser noise

Transmission and noise of a single pixel



Many modes can be better than one

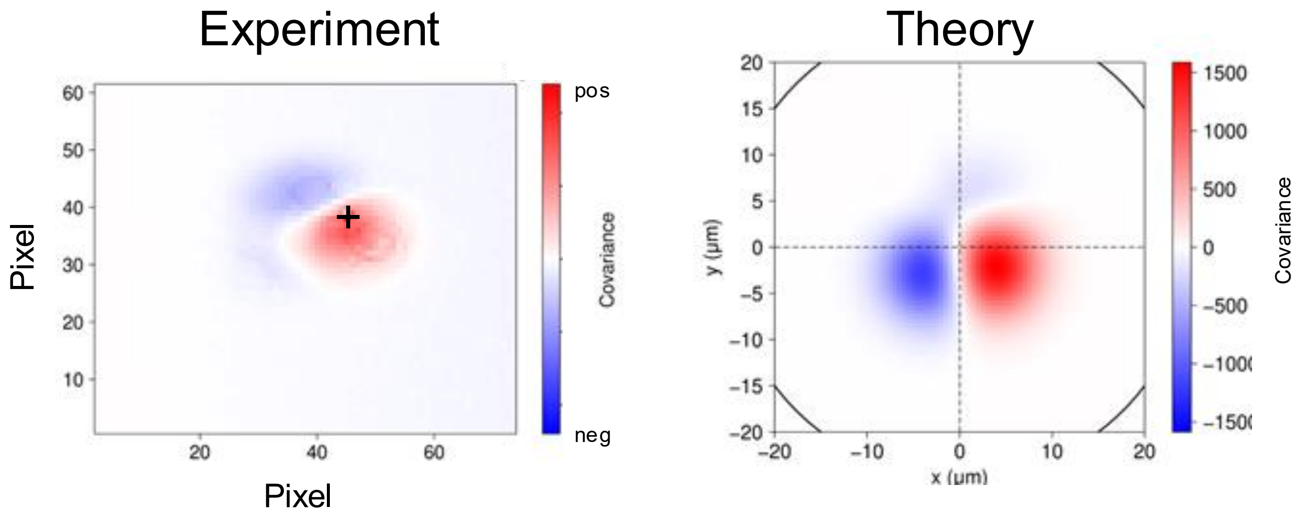


- Access to emergent phenomena
- Breaking the rules of single mode photonics
- Quantum information scales exponentially

Wright, Logan G., William H. Renninger, Demetri N. Christodoulides, and Frank W. Wise. "Nonlinear multimode photonics: nonlinear optics with many degrees of freedom." *Optica* (2022).

Probing spatial correlations

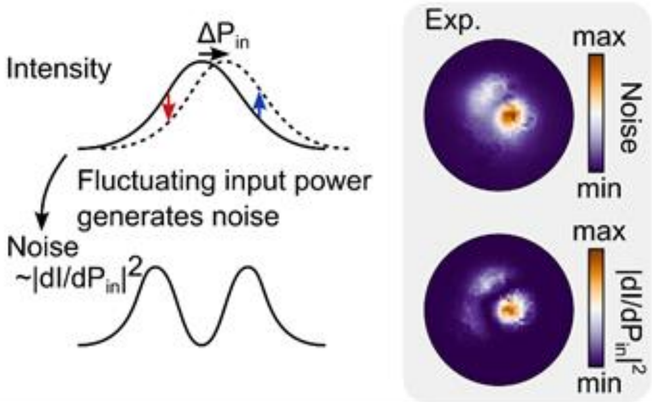
$$\Delta(n_i + n_j)^2 = (\Delta n_i)^2 + (\Delta n_j)^2 + 2\,\mathrm{cov}(n_i, n_j)$$



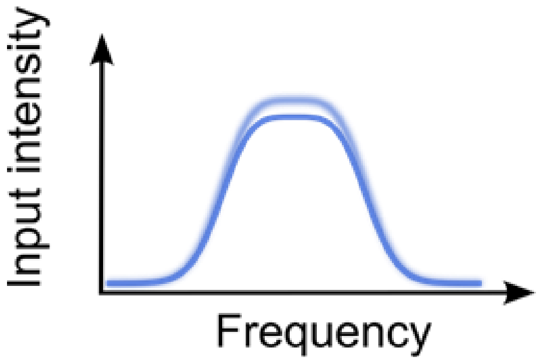
Intensity fluctuations at pixels are correlated with one another

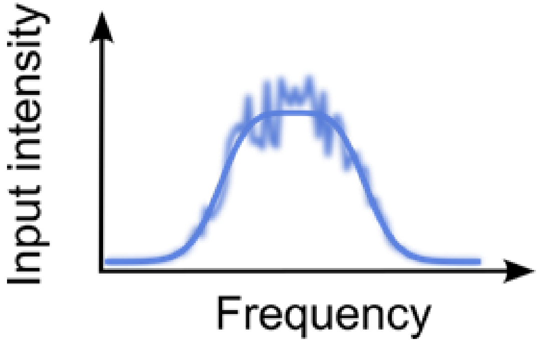
Effective single mode behavior

Single mode behavior

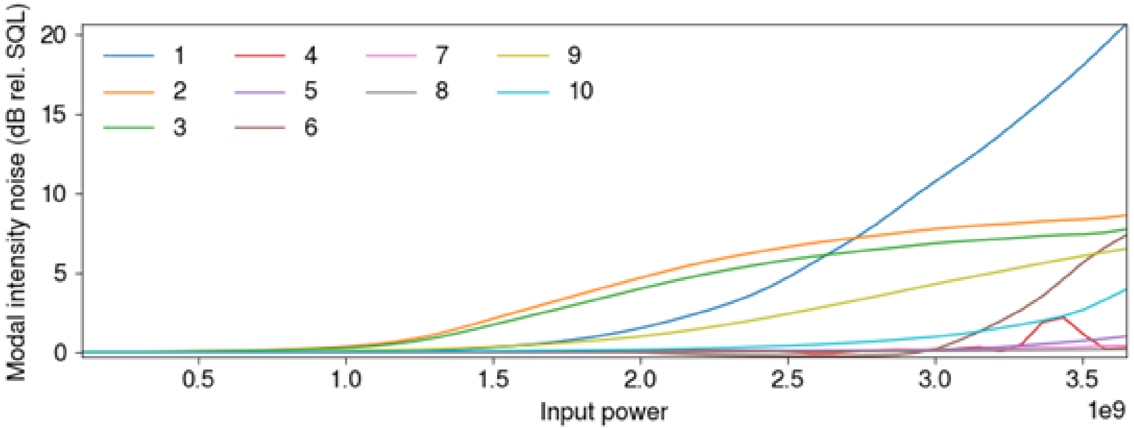


Complex multimode behavior

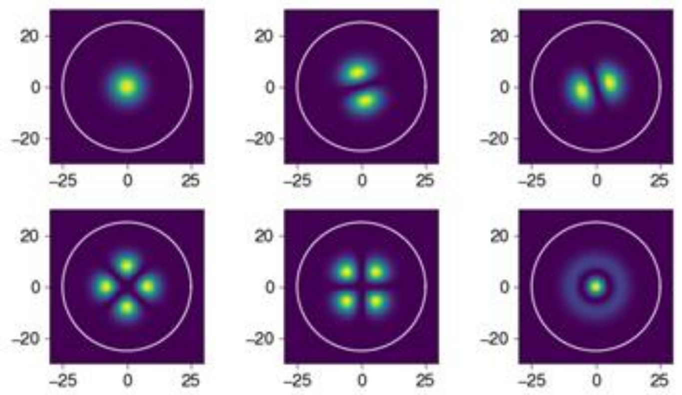




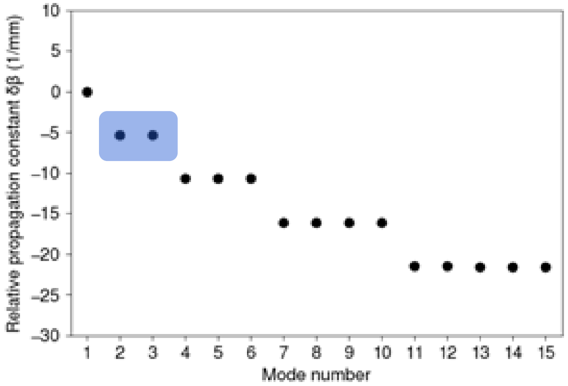
Eigenmode behaviors from theory



Nonlinear propagation in multimode fibers



Spatial modes



Propagation constants

Nonlinear Schrodinger Equation (NLSE)

$$\frac{d\hat{u}_i}{dz} = i\beta_i \hat{u}_i + i \sum_{j,k,l} \gamma_{ijkl} \hat{u}_j \hat{u}_k \hat{u}_l^\dagger$$

Linear phase

Phase modulation, four wave mixing

Linearized quantum fluctuations

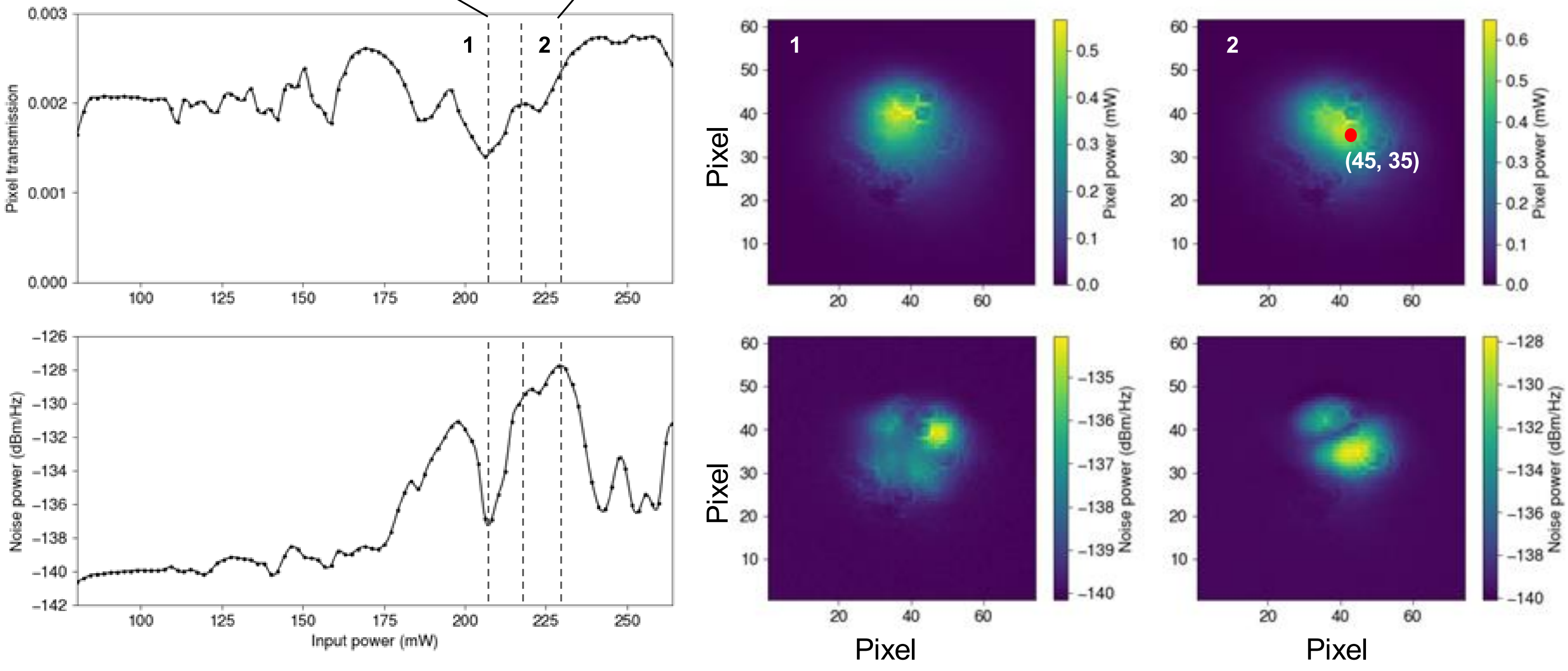
$$\hat{u}_i = \langle \hat{u}_i \rangle + \delta \hat{u}_i$$

$$[\hat{u}_i, \hat{u}_j^\dagger] = \delta_{ij}$$

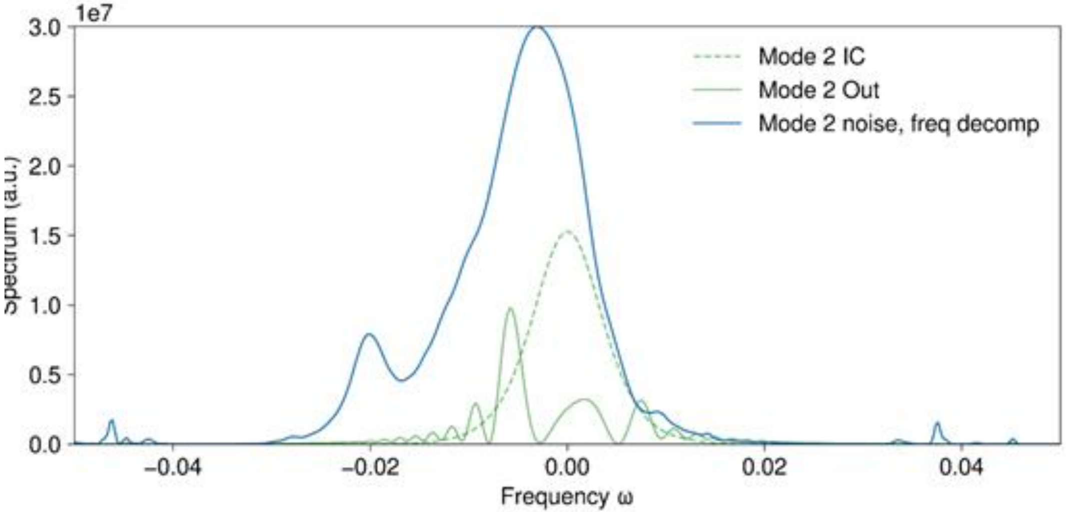
Intensity dependent transmission at a pixel

Dip in transmission, low noise

Rise in transmission, High noise



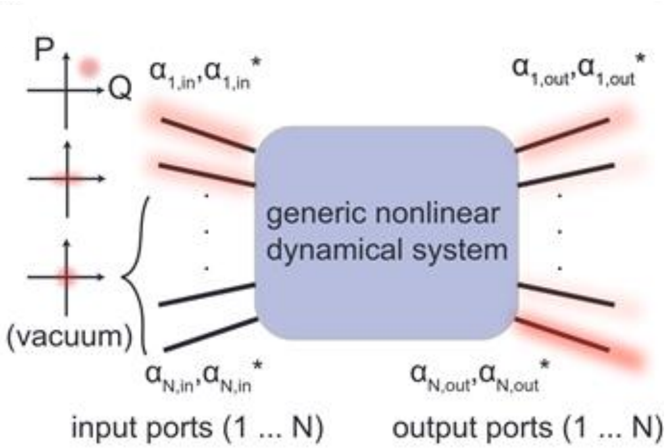
Frequency decomposition of noise

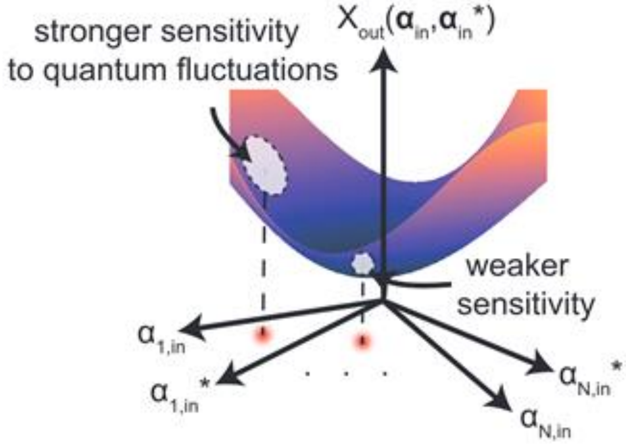


Insights:

- Some frequencies cause more noise than other

Quantum sensitivity analysis





Variance in some observable

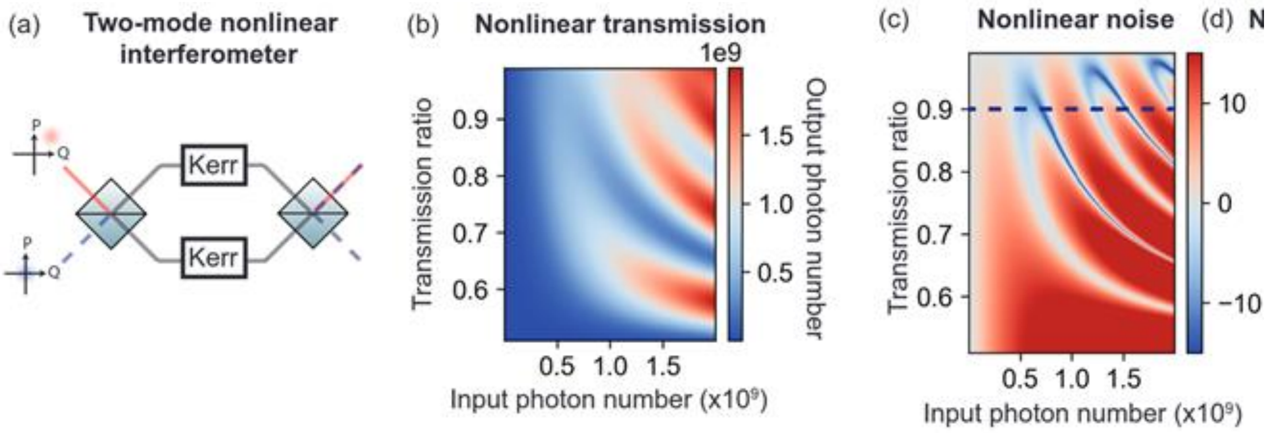
$$(\Delta X)^2 = \sum_l \left| \frac{\partial X}{\partial \alpha_l(0)} \right|^2$$

Sum over all channels

Gradients of observable with respect to all initial conditions

Relies only on deterministic classical simulations

Example of a Kerr sagnac interferometer



$$(\Delta n_{\mathrm{out}})^2 = \left(\frac{\partial n_{\mathrm{out}}}{\partial n_{\mathrm{in}}}\right)^2 (\Delta n_{\mathrm{in}})^2 + \left|\frac{\partial n_{\mathrm{out}}}{\partial \beta_{\mathrm{in}}}\right|^2$$

Intensity gradient Input noise Vacuum contribution

Wavefront shaping to control noise (thy.)