AEP 4400 / 5400 Nonlinear and Quantum Optics

Exam 1

Your name: ______________________________

Question 1: ______________________________ /15

Question 2: ______________________________ /15

Question 3: ______________________________ /32

Question 4: ______________________________ /38

Total: ______________________________ /100

Instructions: You have 90 minutes to complete this exam. Do not start until instructed to do so. There are four questions. You may reference your notes, the course lecture notes, the textbook, and previous homework assignments. You may quote results derived in class, discussion, or problem sets without re-deriving them, unless explicitly asked. Note that for questions which require you to show a result: if the result is given, you may be able to do the subsequent parts of the problem even if you do not show the intended result. **Please turn in the exam sheets, your blue book, and any scrap paper you used.**

I. TRUE/FALSE QUESTIONS [15 POINTS]

(a) [5 points] In a centro-symmetric material, $\chi^{(3)}$ must vanish.
(b) [5 points] For a lossless and instantaneous nonlinear material $\chi^{(2)}_{xyz} = \chi^{(2)}_{xzy}$.
(c) [5 points] Consider an electro-optic crystal of class $\bar{4}2m$ whose extraordinary axis is along the z direction. If light is sent into this crystal with its polarization oriented at an angle 45 degrees to the x-axis, it will experience amplitude modulation upon applying an appropriate time-dependent voltage difference along the z direction.

II. DEGENERATE PARAMETRIC AMPLIFIER [15 POINTS]

Consider the inverse process to second-harmonic generation in which a photon at frequency $2\omega_1$ generates two photons of frequency ω_1. This process is called degenerate parametric down-conversion and can be used to generate light at half the frequency of the input.

(a) [5 points] Write down coupled amplitude equations for the amplitude A_1 of the electric field at frequency ω_1 and the amplitude A_2 of the electric field at frequency $\omega_2 = 2\omega_1$.

(b) [10 points] Assuming phase-matching, find $A_1(z)$ assuming $A_2(z)$ is non-depleted. Both $A_1(0)$ and $A_2(0)$ may be assumed to be nonzero. Hint: one way to go about this is to derive an appropriate second-order differential equation.

III. SPATIAL WALK-OFF IN TYPE-I PHASE MATCHING [32 POINTS]

Consider type-I phase-matching for second-harmonic generation in a negative uniaxial crystal. As described in class, the efficiency of this process is limited by spatial walk-off: the extra-ordinary polarized second-harmonic beam moves in a different direction from the ordinary-polarized fundamental beam (despite their wavevectors being collinear).

(a) [6 points] Show that the time-averaged Poynting vector $\mathbf{S}_1$ for the ordinary-polarized fundamental wave is exactly parallel to $\mathbf{k}_1$ for any level of anisotropy. Recall that the time-averaged Poynting vector associated with an electric field $\mathbf{E}e^{-i\omega t}$+c.c, and its associated magnetic field $\mathbf{H}e^{-i\omega t}$+c.c,is given by $\mathbf{S} = 2\text{Re}\,\mathbf{E}\times\mathbf{H}^*$.

(b) [6 points] The Poynting vector $\mathbf{S}_2$ of the second-harmonic field is *not* parallel to $\mathbf{k}_2$ in an anisotropic material, and in general makes an angle $\theta_{k,S}$ to the wavector. Show however that in the absence of anisotropy, $\theta_{k,S} = 0$.

(c) [6 points] Even in a uniaxial anisotropic medium, there is a special orientation between the wavevector and the c-axis such that the Poynting vector of the second harmonic is parallel to the wavevector (for type-I phase matching in a negative uniaxial crystal). What is that orientation? Draw a sketch which illustrates the c-axis and the wavevector of the second harmonic.

(d) [6 points] For weak birefringence ($|n_o - n_e| \ll n_o$), it can be shown that the angle $\theta_{k,S} = 0$ between the Poynting vector of the second harmonic and the second-harmonic wavevector is

$$\theta_{k,S} \approx \frac{n_o - n_e}{n_o} \sin 2\theta_{k,c}, \tag{1}$$

where $\theta_{k,c}$ is the angle between the second-harmonic wavevector and the c-axis. Suppose the fundamental beam has a transverse extent w (this could be the beam diameter for example). What is the length scale of propagation over which the second-harmonic beam gets separated from the fundamental? This is called the walk-off length. Give an order-of-magnitude estimate for this walk-off length assuming $w = 100\ \mu$m.

(e) [8 points] How does temperature tuning get around the problem of spatial walk-off? In your answer, clearly state how the wavevectors of the fundamental and second harmonic are oriented relative to the c-axis, and the polarizations of the fundamental and second-harmonic. Be sure to clearly explain why walk-off is avoided.

IV. FIFTH-ORDER OPTICAL NONLINEARITY [38 POINTS]

For high enough field amplitude, we would expect components of the electric polarization at order $\chi^{(n)}$ with $n > 3$ to contribute. The goal of this problem is to explore "self-effects" in this situation.

(a) [8 points] Consider a single driving field at frequency ω. Explain why a material will have components of the nonlinear polarization at the same frequency as the driving laser field only for orders $P^{(n)}$ where n is odd. You may consider the material to be instantaneous for the purpose of simplicity (although it is not necessary), and ignore cascading.

(b) [10 points] For an applied laser field, $E(\omega) = E_0 e^{ikz-i\omega t}$ + c.c., write down all polarization contributions up to *fifth* order in the driving field. Pay close attention to the number of permutations of each term. (N.B., there is no need to write down polarization field components at frequencies other than ω.) Assume all polarization fields are parallel to the applied laser field (and it is thus okay to write down scalar expressions).

(c) [10 points] Assuming $\chi^{(5)}$ is purely real, show that the $P^{(5)}$ term from part (b) does not contribute a change to the field amplitude during propagation, and thus its only effect is to cause a nonlinear phase shift. (Note, you may consider this effect in isolation of all other propagation effects such as $\chi^{(1)}, \chi^{(3)}$.) Hint: assume that the frequency-domain Maxwell's equations with the nonlinear polarization admit a complex solution $E(z, \omega) = E_0 e^{ikz}$ and find $k(\omega, |E_0|)$.

(d) [10 points] Starting with Maxwell's wave equation, show that the expression you found for the polarization in (b) leads to a sensible new definition of the index of refraction $n(I) = n_0 + n_2 I + n_4 I^2$, where I is the light intensity. Find an expression for n_4 in terms of $\chi^{(5)}$ and other parameters.