AEP 4230 Exam 1 Practice

Timing: 90 minutes.

I. TRUE/FALSE QUESTIONS (10 POINTS)

No need to explain your answer, just answer true or false.

(a) [3 points] $\int_A^B \frac{dQ}{T} = -\int_B^A \frac{dQ}{T}$ for any process taking thermodynamic state A to thermodynamic state B.

(b) [3 points] In a system at fixed generalized coordinates and temperature, the condition for a process to occur spontaneously is $\Delta S < 0$, where S is the entropy of that system.

(c) [4 points] For a system with one chemical species and two ways of doing work, the dimensionality of the coexistence line between two phases is 2.

II. DO WE HEAT THE ROOM? (10 POINTS)

A physicist and an engineer find themselves in a mountain lodge where the only heat is provided by a large woodstove. The physicist argues that they cannot increase the total energy of the molecules in the cabin, and therefore it makes no sense to continue putting logs into the stove. The engineer strongly disagrees, referring to the laws of thermodynamics and common sense. Who is right? Why do we heat the room? Note: it may be that each person has a point, but you should discuss the relative merits and misconceptions in each perspective. A few sentences should do here.

III. SPRING CYLINDER (15 POINTS)

One part of a cylinder is filled with one mole of a monatomic ideal gas at a pressure of 1 atm and temperature of 300 K. A massless piston separates the gas from the other section of the cylinder which is evacuated but has a spring at equilibrium extension attached to it and to the opposite wall of the cylinder. The cylinder is thermally insulated from the rest of the world, and the piston is fixed to the cylinder initially and then released. After reaching equilibrium, the volume occupied by the gas is double the original. Neglecting the thermal capacities of the cylinder, piston, and spring, find the temperature and pressure of the gas.

IV. MAGNETIZATION OF SPINS (15 POINTS)

Consider N spins with two magnetization states ± 1. They correspond to a magnetic moment $\pm\mu$ in a direction $\hat{z}$. The spins do not interact with each other but are otherwise placed in a magnetic field $\mathbf{B} = B\hat{z}$, and the Hamiltonian is

$$H = (-\mu B)\sum_i \sigma_i. \tag{1}$$

The magnetization is defined as the difference $M = \mu(N_+ - N_-)$, where N_+ is the number of spins pointing upwards ($\sigma = 1$) and N_- the number of spins pointing downwards $\sigma = -1$.

(a) [10 points] Find the magnetization of the system as a function of B and temperature T.

(b) [10 points] Find the magnetic susceptibility as a function of temperature $\chi(T) = \lim_{B\to 0} \frac{\partial M}{\partial B}\Big|_T$.

V. BIRTHDAY PROBLEM (20 POINTS)

(a) [10 points] There are n people in a room. Show that the probability that at least two people have a birthday in common is

$$p = 1 - \frac{D!}{(D-n)!D^n}, \tag{2}$$

where D is the number of days in a year. Ignore the possibility of a leap year, and assume that any person's birthday is equally likely to be at any point of the year.

(b) [5 points] Show that this probability may be approximated as

$$p = 1 - e^{-n}\left(\frac{D}{D-n}\right)^{D-n}. \tag{3}$$

(c) [5 points] Using the approximation $\left(1 - \frac{n}{D}\right)^D \approx e^{-n-\frac{1}{2}\frac{n^2}{d}}$ for $d \ll n$, estimate the number of people required such that the probability of at least two people having the same birthday is $1 - e^{-1} \approx 0.62$.

VI. VAN DER WAALS GAS (30 POINTS)

A monatomic gas satisfies the van der Waals equation:

$$P = \frac{Nk_BT}{V - Nb} - a\frac{N^2}{V^2}, \tag{4}$$

with a and b constants. The gas has a heat capacity $C_V = \frac{3}{2}Nk_B$ in the limit $V \to \infty$.

(a) [7 points] Show that

$$\left.\frac{\partial C_V}{\partial V}\right|_T = 0 \tag{5}$$

(b) [8 points] Use the preceding result to determine the entropy $S(T, V)$ up to an overall constant. Hint: when determining the path of integration, consider that the only place where you know anything specific about the heat capacity is at $V = \infty$.

(c) [8 points] Determine the internal energy $E(T, V)$ to within an additive constant.

(d) [7 points] What is the final temperature when the gas is adiabatically compressed from volume V_1 to volume V_2? The initial temperature is T_1.